Slide 1

Administrivia

- Midterm grades mailed earlier today, including letter-grade estimate. Things to note if you're not happy with yours:
  - In computing the final grade, I'll drop the lowest quiz score. I didn't do that here.
  - Letter grades are conservative estimates.
  - There are still lots of points in play, and there will likely be a possibility of extra-credit points at the end of the term.

Slide 2

Minute Essay From Last Lecture

- Question: If a password is at least 5 characters and no more than 8, where a character is either a digit or a lowercase character, how many possible passwords are there? (Okay to just write down an expression and not simplify — e.g., 10^4.)
- Answer?

Slide 3

Counting, Recap/Review

- Multiplication principle — if there are N ways to do one thing, and M ways to do another, there are $N \times M$ ways to do first one and then the other.
- Addition principle — if there are N ways to do one thing, and M ways to do another, there are $N + M$ ways to do one or another.
- Can combine these in interesting and effective ways. Recall examples from earlier class. One more example — section 3.2 problem 58.
- Decision trees also sometimes useful. Recall example from earlier class (sequences of heads and tails).

Slide 4

Principle of Inclusion/Exclusion

- Motivating(?) example:

  You take a poll of how many people support propositions A and B. You find that 10 of them support A, 20 support B, and 5 support both A and B. How many support either A or B?
- Using set notation, with $|S|$ meaning the number of elements in S:
  Given $|A| = 10$, $|B| = 20$, and $|A \cap B| = 5$,
  what is $|A \cup B|$?
- We can use the addition principle to derive

$$|A \cup B| = |A| + |B| - |A \cap B|$$

  (Intuitive idea is that we count everything in both sets, and in doing that we count some things twice, so we must correct.)

  Apply to example.

Slide 5

Principle of Inclusion/Exclusion, Continued

- What if there were three propositions? for example,

  10 people support A

  20 support B

  15 support C

  5 support both A and B

  5 support both A and C

  5 support both B and C

  2 support all three

- Can we derive a rule for three sets? (Yes, from the rule for two sets and properties of set union and intersection.)

Slide 6

Principle of Inclusion/Exclusion, Continued

- Rule for three sets is

  $$|A \cup B \cup C| = |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

  Intuitive idea:

  Count all the A's, all the B's, all the C's.

  A&B's, B&C's, and A&C's have been counted twice; A&B&C's have been counted three times.

  Subtract counts of A&B's, B&C's, and A&C's; now A&B&C's have been counted zero times.

  Add count of A&B&C's.

- There's a pattern here, captured in general form of rule (p. 205).

Slide 7

Pigeonhole Principle

- Idea is that if you have n items placed in k bins, and $n > k$, then at least one bin has more than one item.

  Converse is that if no bin contains more than one item, n can be at most — what?

- Example — section 3.3 problem 17.

Slide 8

Minute Essay

- An easy one: How was your spring break?