**Slide 1**

## Administrivia

- Reading for Wednesday was wrong — should be 3.5 not 3.6. Corrected now.

**Slide 2**

## Finite Probability — Review/Recap

- Probability of event $E$ in sample space $S$: $P(E) = \frac{|E|}{|S|}$.
- Conditional probability of $E_2$ given $E_1$: $P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$.
  Equivalent to $\frac{|E_1 \cap E_2|}{E_1}$.
- For disjoint events $E_1$ and $E_2$, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$. Follows from definition, facts about sets.
- For independent events (defined in terms of conditional probability), $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$.

Slide 3

## Examples

- If a fair coin is tossed four times, what's the probability of getting four heads? What's the probability that the last toss is a head given that the first three are heads?
- In a group of $n$ people, what's the probability that at least two people have the same birthday?

Slide 4

## Expected Value

- You probably know about computing weighted averages — from classes in which your grade is computed as, e.g., 50% exams, 20% homework, etc.
- "Expected value" is a generalization of this: Given a sample space $S$, a "random variable" $X$ (function from $S$ to $\mathbb{R}$), and a probability distribution $p$, define expected value of $X$ thus:

  $$E(X) = \sum_{x \in S} X(x)p(x)$$

  Intuitive idea — "average" value, where the average is weighted by how likely the different values are.

Slide 5

## Average-Case Analysis of Algorithms

- Previously we talked about estimating worst-case execution time of algorithms — amount of "work" as a function of input size.
- We could also talk about average-case amount of work, based on idea of expected value: Sample space is set of all possible inputs. For input $x$, $X(x)$ is the amount of work for $x$ and $p(x)$ is the probability of $x$.

  Example — example 68 in textbook.

Slide 6

## Binary (and other) Relations

- Idea of a binary relation is to express relationship between pairs of elements of a set. Some interesting special cases:
  - Partial orderings — useful in working out how we could put things "in order", e.g., a set of tasks with (some) ordering dependencies.
  - Functions (of 1 variable).
- Generalization — "$n$-ary relation", also with interesting special cases:
  - Functions of more than 1 variable.
  - Relational databases.

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## Binary Relations

- Formal definition: A binary relation $\rho$ on a set $S$ is a subset of $S \times S$. Usually this set is defined by some property of interest. For $a, b \in S$, we write $a \rho b$ iff (if and only if) $(a, b)$ is in this subset.
- Examples:
  - $S$ is people in the world; $x \rho y$ iff $x$ and $y$ are siblings.
  - $S$ is integers; $x \rho y$ iff $x < y$.
  - $S$ is integers; $x \rho y$ iff $y$ is a multiple of $x$.
  - $S$ is integers; $x \rho y$ iff $y = x^2$.
- Notice that for a given relation $\rho$ and element $x$, there can be any number (including zero) of $y$'s such that $x \rho y$ and any number (including zero) of $y$'s such that $y \rho x$.
- Next step will be to define "interesting" properties of relations.

Slide 8

## Minute Essay

- If a fair coin is tossed four times, what's the probability of getting two heads and two tails given that there's at least one head and at least one tail?