Administrivia

- None.

Slide 1

Minute Essay From Last Lecture

- Question: If we define a relation ρ on the students in this class, such that $x\ \rho\ y$ iff x and y are sitting in the same row:

  Is ρ reflexive? symmetric? transitive? antisymmetric?

- Answer?

Slide 2

Slide 3

Closures

- Last time we talked about several properties a relation can have — reflexivity, symmetry, etc.
- We can also talk about the “closure” of a relation with respect to a property — the smallest superset of the relation that has the property.
- Example: Define relation ρ on integers such that $x \ \rho \ y$ iff $y = x + 1$. What is the transitive closure of ρ?

Slide 4

Uses of Partial Orderings

- As mentioned last time, a partial ordering (reflexive, symmetric, transitive relation — think “generalized $\leq$”) can express ordering constraints among tasks.
- We'll look at two applications — PERT charts and topological sorting.

PERT Charts

Slide 5

- (PERT is "Program Evaluation and Review Technique".)
- Idea is to start with a set of tasks, each of which can have others as prerequisites (i.e., a partial ordering), and express these relationships graphically, and also include time to complete each task. From the diagram, can then determine minimum time to complete all tasks, "critical path".
- Example — practice problem 17 in text.

Topological Sorting

Slide 6

- Idea here is to take a partial ordering and find a way to extend it to a "total" ordering (i.e., add pairs so that for every x and y either $x \; \rho \; y$ or $y \; \rho \; x$. How is this useful? e.g., find a way to "schedule" interdependent tasks.
- Notice that there could be more than one way to do this for a given partial ordering.

Slide 7

Topological Sorting, Continued

- Algorithm for finding *a* way to extend a partial ordering — “topological sort”:
- Start with set S and partial orderig ρ on S. Idea is to turn S into a sequence $x_1, x_2, \ldots$ such that $(x_i \; \rho \; x_j) \;\rightarrow\; (i \leq j)$.
- The algorithm might look like this in pseudocode:

  while (S not empty)
  pick a minimal element x in S
  make it the next element of the sequence and remove it from S
  end while

- Does this work? i.e., does it produce an ordering that extends ρ? True if we can be sure that for x and y with $x \; \rho \; y$ x is picked before y.
- Try this on previous example . . .

Slide 8

Minute Essay

- None — quiz.
- Reminder — Homework 6 due by 5pm.