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Probability — Not-Equally-Likely Outcomes

- One approach extend previous definition (size of "event" divided by size of sample space) by adding duplicates to sample space for outcomes that are more likely.
- Another approach "probability distribution": For each x in sample space S, assign x a probability p(x), such that

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$$0 \le p(x) \le 1$$
, for all $x \in S$
$$\sum_{x \in S} p(x) = 1$$

• Now for event E ($E \subseteq S$), we have

$$P(E) = \sum_{x \in E} p(x)$$

• Note that equally-likely-outcomes definition is a special case of the above.

Conditional Probability

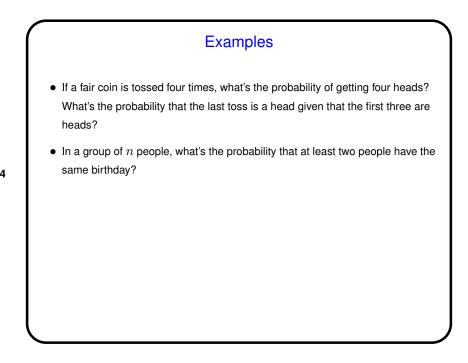
• We could also consider, for two possibly related events E_1 and E_2 , how likely it is that E_2 happens given that E_1 has happened — "conditional probability":

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

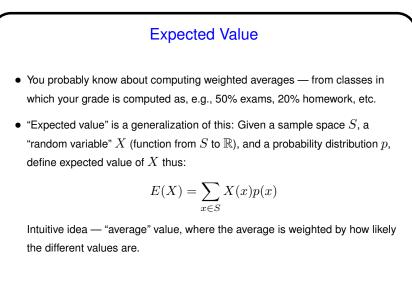
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Intuitive idea is that here the "sample space" is limited to E_1 .

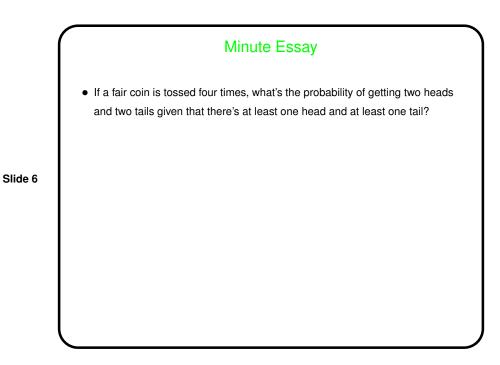
- If it turns out that $P(E_2|E_1) = P(E_2)$, we call E_1 and E_2 "independent events". In this case, we can derive that $P(E_1 \cap E_2)$ is what?
- Notice resemblance between this and multiplication principle, and between rule last time for $P(E_1 \cup E_{@})$ and addition principle.



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Minute Essay Answer • Sample space has 16 elements (different outcomes of flipping coin four times). If E_1 includes outcomes with at least one head and at least one tail, $P(E_1)$ is 14/16, because E_1 is all of the sample space except the "all heads" and "all tails" outcomes. If E_2 includes outcomes with two heads and two tails, $P(E_2)$ is 6/16, because there are C(4, 2) = 6 ways to choose the two tosses that come up heads. $E_1 \cap E_2$ is just E_2 . So from definition, $P(E_2|E_1) = (6/16)/(14/16)$, i.e., 6/14.

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