Slide 1

Administrivia

- Reminder: Quiz 1 Wednesday. Okay to use textbook and any notes. Questions will be about propositional logic.
- Homework 1 due by 5pm today.
- Homework 2 on Web; due next Monday.

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Propositional Logic Example, Continued

- (Finish section 1.2 problem 34.)

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Why Predicate Logic?

- Propositional logic captures some of what we need to talk about things logically, but not everything.
- Example from classical logic:

  "All humans are mortal. Socrates is human. Therefore Socrates is mortal."

  No way to express this in propositional logic.
- What we want to add is some way to express the idea of something being true "for all x" or "for at least one x".

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Predicates

- Define "predicate" — boolean-valued function of one or more variables.
- Examples with integer variables:

  $P(x) = (x > 0)$

  $Q(x, y) = (x < y)$
- Examples with people variables:

  $P(x)$ means "x is a student in CSCI 1323".

  $Q(x, y)$ means "x is taller than y".

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Quantifiers

- Universal quantification: $(\forall x)P(x)$ means "for all x, $P(x)$ is true."
- Existential quantification: $(\exists x)P(x)$ means "there exists an x such that $P(x)$ is true."
- How to decide whether such a statement is true? For propositional-logic connectives, we could write down a truth table for different values of the formulas being connected. That won't work here. (Why?)
- Instead, notion of a "domain of interpretation" — (non-empty) range of values for the variable, definition of predicate(s).

  $(\forall x)P(x)$ means — ?

  $(\exists x)P(x)$ means — ?

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A Few More Definitions

- Define "variables" (usually write them x, y, etc.) and "constants" (usually write them a, b, etc.) — elements of domain of interpretation.
- "Free variables" are those not within scope of a quantifier — e.g., x but not y in $(\forall y)P(x, y)$.
- Notice that we can change the variable in a quantification — it's a "dummy variable" – except we can't duplicate another variable.
- As in propositional logic, can define notion of well-formed formula (wff) — "sensible" combination of predicates, quantifiers, connectives from propositional logic, and parentheses.
- How to express "All men are mortal", etc?

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Interpretations

- Expressions involving predicates are true/false depending on "interpretation" (analogous to assigning values to statements in propositional logic):
  - Domain of the interpretation (must not be empty).
  - Assignment of a property of objects in the domain to each predicate.
  - Assignment of a particular object to each constant symbol.
- Given an interpretation and an expression, we can (usually) compute a value for it. (What if there's at least one free variable?)

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Interpretations — Example

- Suppose the domain is the integers, and $Q(x)$ means "x has an integer square root".
- What is the "truth value" of the following?
  - $Q(4)$
  - $Q(2)$
  - $(\forall x)Q(x)$
  - $(\exists x)Q(x)$
  - $Q(4) \ \lor \ Q(2)$
  - $Q(c)$
  - $Q(x)$

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English to Formulas

- Given people as a domain and predicates
  - $C(x)$ meaning "x is a CS student"
  - $D(x)$ meaning "x must pass CSCI 1323 to graduate"
  - $B(x)$ meaning "x is a business major"
  - $M(x)$ meaning "x likes math"
- Translate:
  - "'All CS majors must pass CSCI 1323 to graduate."
  - "Some CS majors are business majors."
  - "Some CS majors like math."
  - "Not all CS majors like math."

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Minute Essay

- Consider formulas $Q(a)$, $Q(b)$, $(\forall x)Q(x)$. Tell me whether each is true or false for the following interpretations.
- Interpretation 1: domain of interpretation is the integers, $a = 1$, $b = 2$, and $Q(x)$ means "$2x$ is an even integer".
- Interpretation 2: domain of interpretation is the rational numbers, $a = 1/2$, $b = 1$, and $Q(x)$ means "$2x$ is an even integer".

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Minute Essay Answer

- Interpretation 1: domain of interpretation is the integers, $a = 1$, $b = 2$, and $Q(x)$ means "$2x$ is an even integer".

  $Q(a)$ true, $Q(b)$ true, $(\forall x)Q(x)$ true.

- Interpretation 2: domain of interpretation is the rational numbers, $a = 1/2$, $b = 1$, and $Q(x)$ means "$2x$ is an even integer".

  $Q(a)$ false, $Q(b)$ true, $(\forall x)Q(x)$ false.