Slide 1

Administrivia

- Homework 3 deadline extended to next Wednesday.
- "Useful links" Web page has links to more examples of mathematical induction.

Slide 2

Mathematical Induction — Overview

- Basic idea is to prove something true for all integers greater than some base value (usually 0 or 1) in two steps:
  - Base case — prove directly for smallest value.
  - Inductive step — prove that if true for k (first principle), or all numbers from base case through k (second principle), then also true for $k + 1$.
- Works because the base case gives you a starting point, and the inductive step can be used to build up a sequence of implications, and then from propositional logic . . .
- If you know about recursive functions/procedures: Inductive step is similar to non-base case — break up problem for $k + 1$ into "smaller problems" that you can "solve" with the inductive hypothesis.

Slide 3

First Principle of Mathematical Induction

- We can prove that $P(k)$ is true for all integers $k \geq N$ (often N is 0 or 1, but not always) if we can show:
  - Base case: $P(N)$
  - Inductive step: For $k \geq N$, $P(k) \rightarrow P(k+1)$
    That is: Assume $P(k)$ and $k \geq N$ ("inductive hypothesis"), and show that then $P(k+1)$
- For readability/clarity, make this explicit, especially what you assume / have to show for inductive step.
- Works because we have $P(N)$ and then a chain of implications: $P(N) \rightarrow P(N+1), P(N+1) \rightarrow P(N+2), \ldots$

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First Principle of Mathematical Induction — Examples

- Example: Show that for $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

- Example: Show that for $n \geq 1$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

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Second Principle of Mathematical Induction

- Can also show that $P(k)$ is true for all integers $k \geq N$ (often N is 0 or 1, but not always) if we can show that:
  - Base case: $P(N)$
  - Inductive step: For $k \geq N$, $((N \leq r \leq k) \rightarrow P(r)) \rightarrow P(k+1)$
    That is: Assume that $P(r)$ holds for all integers r with $N \leq r \leq k$, and that $k \geq N$ (“inductive hypothesis”), and show that then $P(k+1)$
- For readability/clarity, again make this explicit . . .
- Notice — inductive hypothesis here is more complicated, but gives you more to work with.
- Works because we have $P(N)$ and then a chain of implications: $P(N) \rightarrow P(N+1), P(N) \wedge P(N+1) \rightarrow P(N+2), \ldots$

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Second Principle of Mathematical Induction — Example

- Consider a perforated sheet of stamps. How many “tear into two sheets” operations are needed to produce single stamps?
- Conjecture, based on some examples — ?
- Can prove with second principle . . .

Minute Essay

- None — quiz.

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