Slide 1

Administrivia

- Reminder: Homework 8 due Wednesday.
- Reminder: Quiz 5 Wednesday. Questions from 3.4, 3.5.

Slide 2

Probability — Equally-Likely Outcomes (Review)

- Basic definition: If S ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and E ("event") is a subset of S, then we define the probability of E as

$$P(E) = \frac{|E|}{|S|}$$

- Note that $0 \leq P(E) \leq 1$. (Why?) When is $P(E) = 0$? When is $P(E) = 1$?
- Note that we can apply anything we know about sizes of sets. (E.g., if E_1 and E_2 are disjoint, what is $P(E_1 \cup E_2)$ in terms of $P(E_1)$ and $P(E_2)$?)

Slide 3

Probability — Not-Equally-Likely Outcomes

- One approach — extend previous definition (size of "event" divided by size of sample space) by adding duplicates to sample space for outcomes that are more likely.
- Another approach — "probability distribution": For each x in sample space S, assign x a probability $p(x)$, such that

$$0 \leq p(x) \leq 1, \text{for all } x \in S$$

$$\sum_{x \in S} p(x) = 1$$

- Now for event E ($E \subseteq S$), we have

$$P(E) = \sum_{x \in E} p(x)$$

- Note that equally-likely-outcomes definition is a special case of the above.

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Conditional Probability

- We could also consider, for two possibly related events E_1 and E_2, how likely it is that E_2 happens given that E_1 has happened — "conditional probability":

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

Intuitive idea is that here the "sample space" is limited to E_1.

- If it turns out that $P(E_2|E_1) = P(E_2)$, we call E_1 and E_2 "independent events". In this case, we can derive that $P(E_1 \cap E_2)$ is — what?
- Notice resemblance between this and multiplication principle, and between rule for $P(E_1 \cup E_2)$ and addition principle.

Examples

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- If a fair coin is tossed four times, what's the probability of getting four heads? What's the probability that the last toss is a head given that the first three are heads?

Expected Value

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- You probably know about computing weighted averages — from classes in which your grade is computed as, e.g., 50% exams, 20% homework, etc.
- "Expected value" is a generalization of this: Given a sample space S, a "random variable" X (function from S to $\mathbb{R}$), and a probability distribution p, define expected value of X thus:

$$E(X) = \sum_{x \in S} X(x)p(x)$$

  Intuitive idea — "average" value, where the average is weighted by how likely the different values are.

Average-Case Analysis of Algorithms

- Previously we talked about estimating worst-case execution time of algorithms — amount of “work” as a function of input size.
- We could also talk about average-case amount of work, based on idea of expected value: Sample space is set of all possible inputs. For input x, $X(x)$ is the amount of work for x and $p(x)$ is the probability of x.

  Example — example 68 in textbook.

Slide 7

Minute Essay

- If a fair coin is tossed four times, what’s the probability of getting two heads and two tails given that there’s at least one head and at least one tail?

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Minute Essay Answer

- Sample space has 16 elements (different outcomes of flipping coin four times).

  If E_1 includes outcomes with at least one head and at least one tail, $P(E_1)$ is 14/16, because E_1 is all of the sample space except the "all heads" and "all tails" outcomes.

  If E_2 includes outcomes with two heads and two tails, $P(E_2)$ is 6/16, because there are $C(4, 2) = 6$ ways to choose the two tosses that come up heads.

  $E_1 \cap E_2$ is just E_2.

  So from definition, $P(E_2|E_1) = (6/16)/(14/16)$, i.e., 6/14.