Administrivia

Slide 1

- Reminder: Quiz 6 Wednesday. Likely topics — material about relations and functions from chapter 4.
- In the reading for today (5.1), it's okay to skim/skip the material on planar graphs (pp. 352–356).

Order of Magnitude of Functions, Recap

Slide 2

- Formal definition:

  Write $f = \Theta(g)$ to mean that f and g have the same order magnitude.

  Define to be true iff there are positive constants n_0, c_1, c_2 such that for all $x \geq n_0$

  $$c_1 g(x) \leq f(x) \leq c_2 g(x)$$

  In other words, these functions are roughly proportional to each other.
- Applying definition formally is possible, but lots of tedious algebra. Examples in text.

Slide 3

"Big-O Notation"

- The $O(f(N))$ you see in computer science is similar, but it's a "less than or equal" rather than a "strictly equal" — i.e., $f(N) = O(g(N))$ means f's order of magnitude is no bigger than g's (and might be less).

  Formally, true iff there are positive constants n_0 and c such that for all $x \geq n_0$

$$f(x) \leq cg(x)$$

- Interesting (?) to observe that Θ is an equivalence relation, and O is a partial ordering.

Slide 4

Order of Magnitude of Functions, Continued

- So we have a way to compare orders of magnitude of functions, with an "equals" (Θ) and a "less-than-or-equal-to" (O).
- In general, function's order of magnitude determined by fastest-growing term. Some categories of interest:
  - x^2 grows faster than x, x^3 faster than x^2, etc. x^2 and cx^2 "the same".
  - $\log_b x$ grows more slowly than x.
  - b^x grows faster than all polynomials.
  - x^x grows faster than all b^x.

Graphs — Overview

Slide 5

- In some contexts, "graph" means a plot of a function, other pictorial representation of data.
- In other contexts, it's an abstract idea meant to represent relationships among a set of things. Examples:
  - Hasse diagrams of chapter 4.
  - Airline route maps.
  - Simplified maps showing driving distances between cities.

  Common idea — set of things (set elements, cities) and a notion that some pairs of them are connected somehow. Details we don't care about have been "abstracted out".

Graphs — Definition

Slide 6

- Formal definition (undirected graph):
  - Nonempty set N of nodes (vertices).
  - Set A of arcs (edges).
  - Function $f : A \rightarrow \{\{x, y\} | x, y \in N\}$ (unordered pairs of nodes).

  Notice that we can have "loops" and also "parallel arcs".
- Variations/extensions:
  - Directed graph — edges are ordered pairs (i.e., "one-way").
  - Labeled graph — each vertex has some associated info ("label").
  - Weighted graph — each edge has some associated info ("weight").

Graphs — Terminology

Slide 7

- Adjacent nodes (arc from one to the other).
- Loop, parallel arc.
- Simple graph (no loops or parallel arcs).
- Complete graph (every pair of nodes adjacent).
- Path (sequence of arcs), connected graph.
- Cycle, acyclic graph.

Isomorphic Graphs

Slide 8

- What we care about is the relationship between nodes and arcs, not exact visual representation.
- Can formalize this as "isomorphism" — two graphs are isomorphic if one is just a "relabeling" of the other.
- Formal definition is in terms of one-to-one functions, one from nodes of graph G_1 to nodes of graph G_2 and one from arcs of graph G_1 to arcs of graph G_2. Idea is that if an arc connects two nodes in G_1, the corresponding arc in G_2 connects the corresponding nodes.

Computer-Friendly Representation of Graphs

- For humans, representing graphs pictorially usually works well. For computers, other representations work better.
- Key idea is to come up with a way to represent the essential information — set of nodes and which ones are connected.

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Adjacency Matrices

- Idea is to put the n nodes in some (arbitrary) order and define an n-by-n matrix A such that A_{ij} is the number of arcs connecting node i and node j.
- For an undirected graph, what property does this matrix have? that it might or might not have for a directed graph?
- Variation: For a weighted graph with no parallel arcs, we could let A_{ij} be the weight of the arc from node i to node j.

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Adjacency Lists

- Idea is to again put n nodes in some arbitrary order, but rather than a matrix define an array of n lists, one for each node, with the list for node i containing all nodes j that are adjacent to node i. Parallel arcs mean "duplicate" entries.

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Adjacency Matrix Versus Adjacency List

- Which uses less space?
- Which makes it faster to answer the question "is node i adjacent to node j?"

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Minute Essay

- Which of the following functions are $O(N^2)$?

  $g(N) = 100N^2 + N - 1000$

  $h(N) = N^3$

- Which of the following functions are $O(2^N)$?

  $f(N) = 2^N - 5$

  $h(N) = N!$

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Minute Essay Answer

- $O(N^2)$?

  $g(N) = 100N^2 + N - 1000$ — yes

  $h(N) = N^3$ — no

- $O(2^N)$?

  $f(N) = 2^N - 5$ — yes

  $h(N) = N!$ — no