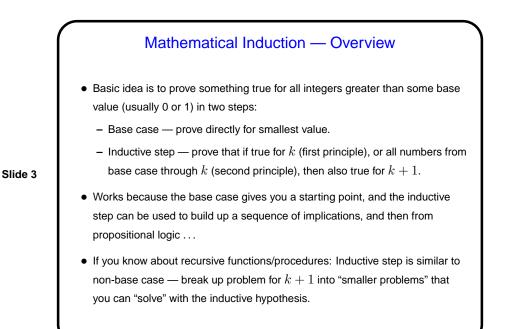


Slide 1





First Principle of Mathematical Induction

- We can prove that P(k) is true for all integers  $k \geq N$  (often N is 0 or 1, but not always) if we can show:
  - Base case: P(N)
  - Inductive step: For  $k \ge N$ ,  $P(k) \rightarrow P(k+1)$
  - That is: Assume P(k) and  $k \geq N$  ("inductive hypothesis"), and show that then P(k+1)
- For readability/clarity, make this explicit, especially what you assume / have to show for inductive step.
- $\bullet\,$  Works because we have P(N) and then a chain of implications:

$$P(N) \rightarrow P(N+1), P(N+1) \rightarrow P(N+2), \dots$$

Slide 4

First Principle of Mathematical Induction — Examples

• Example: Show that for  $n \ge 1$ ,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Slide 5

Example: Show that for 
$$n \geq 1$$
,

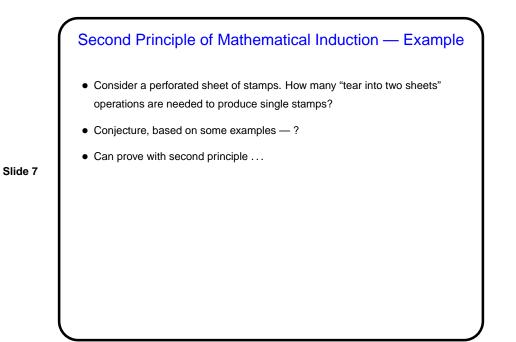
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

## Second Principle of Mathematical Induction

- Can also show that P(k) is true for all integers  $k \ge N$  (often N is 0 or 1, but not always) if we can show that:
  - Base case: P(N)
  - Inductive step: For  $k \ge N$ ,  $((N \le r \le k) \rightarrow P(r)) \rightarrow P(k+1)$ That is: Assume that P(r) holds for all integers r with  $N \le r \le k$ , and that  $k \ge N$  ("inductive hypothesis"), and show that then P(k+1)
- For readability/clarity, again make this explicit ...
- Notice inductive hypothesis here is more complicated, but gives you more to work with.
- $\bullet\,$  Works because we have P(N) and then a chain of implications:

$$P(N) \ \rightarrow \ P(N+1), P(N) \ \land \ P(N+1) \ \rightarrow \ P(N+2), \ldots$$

Slide 6



Minute Essay • None — sign in. Slide 8