Slide 1

Administrivia

- Homework 3 deadline moved to next Tuesday.

Slide 2

Minute Essay From Last Lecture

- Which of the “at the board” approaches seems better?
- No overwhelming majority for either one. Some other alternatives suggested (e.g., small groups, possibly assign problems so each problem gets done by at least one group).

Slide 3

Review/Recap/Overview — Doing (Less Formal) Proofs

- In chapter 1, proofs were like the ones you might have done in a geometry class — very structured, well-defined rules, like a game with a finite number of allowed moves.
- In chapter 2, we use some of what we learned (e.g., De Morgan's law), but proofs are less formal. Easier in that there's less detail; more difficult in that what's allowed is not so well-defined.
- Focus is meant to be more on "proof obligations" and structure of proof than on details.

  (E.g., review/recall wording of minute-essay question (2/07) about proving that there is no largest prime.)

Slide 4

Mathematical Induction — Review/Recap

- Questions usually phrased as "prove that $P(n)$ is true for all integers $\geq n_0$", where $P(n)$ is some statement about n (equation, not formula).
- Two "proof obligations":
  - Base case — usually just n_0, but sometimes must include few numbers right after n_0 as well. (e.g., Example 24 in section 2.2).
  - Inductive step. Notice that *what you are proving is an implication*.
- Why this works — you are proving base cases and a rule for constructing implications, after which you can use universal instantiation and *modus ponens* to get results for non-base cases.

Slide 5

Mathematical Induction — Inductive Step Hints

- What generally works, assuming inductive hypothesis is equation ($f(k) = g(k)$):
  - Write down one side of equation to be proved ($f(k + 1)$).
  - Rewrite it so it somehow includes $f(k)$.
  - Replace $f(k)$ with $g(k)$, then do algebra to show the whole expression equals $g(k + 1)$.
- If proving an inequality, often helpful to use the fact that if $x \leq y$ and $y \leq z$, then $x \leq z$.

Slide 6

Examples — Review

- Section 2.2 problem 31 revisited (we were meant to notice problem 28!).
- "Examples at the board" from last time, revisited.

Slide 7

Recursion and Recursive Definitions

- Idea of recursion closely related to idea of induction — "build on previous smaller cases".
- First look at recursive definitions. To define something recursively:
  - Define one or more "base cases".
  - Define remaining cases in terms of other ("smaller") cases.

Slide 8

Recursive Definitions — Sequences

- A silly example:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  Try writing down some terms.

- Another example:

$$\begin{aligned} S(1) &= 1 \\ S(2) &= 1 \\ S(n) &= S(n-2) + S(n-1), \quad \text{for } n > 2 \end{aligned}$$

  Try writing down some terms. Anyone recognize this one?

Slide 9

Recursive Definitions — Sets

- Example — could define the set of "integer arithmetic expressions" like this:
  - Integers are expressions.
  - If E and F are integer arithmetic expressions, so are $(E + F)$, $(E - F)$, $(E \times F)$, and (E/F).

  Examples?

  Notice that this allows us to generate only "sensible" expressions. Notice also that it's a bit more restrictive than we might like.
- We could write similar definitions for the wffs of propositional and predicate logic.
- Notice: To claim that something is in the set you need to be able to show that it's either a base case or can be obtained from a base case by applying one of the "rules" that define the set.

Slide 10

Minute Essay

- None — quiz.