Administrivia

- Quiz 3 Tuesday. Topics from 2.1 through 2.3 (proofs, proofs by induction).
- Next homework on the Web late today or early tomorrow. Due next Thursday.
- (Not much to say about minute essay from last time — *almost* everyone got it right.)

Slide 1

Solving Recurrence Relations — Review/Recap

- Goal is to take a recursive definition of a sequence and come up with a "closed-form" (non-recursive) definition of the same sequence.
- One method is what textbook calls "expand, guess, verify".
- No other general method (that I know of!), but formulas for some common special cases. We looked at one last time — if the recursive part of the definition is

$$S(n) = cS(n-1) + g(n)$$

the solution is

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n}(c^{n-i}g(i))$$

Slide 2

Slide 3

Another Special Case

- Another case for which there's a formula — two base cases, and the recursive part of the definition depends on the previous two elements ("second-order") in a simple way:

$$S(n) = c_1 S(n-1) + c_2 S(n-2)$$

- For problems that fit this case, solution is more complicated (the "guess" part of getting it seems to involve a rabbit and a hat, though verifying it is relatively straightforward). (Next slide.)

Slide 4

Another Special Case — Solution

- First find roots r_1 and r_2 of

$$t^2 - c_1 t - c_2 = 0$$

- and then (if $r_1 \neq r_2$) find p and q to solve

$$p + q = S(1)$$

$$pr_1 + qr_2 = S(2)$$

- and then the formula is

$$S(n) = pr_1^{n-1} + qr_2^{n-1}$$

Slide 5

Yet Another Special Case

- One more case for which there's a formula is one of interest in analysis of algorithms, especially those that take a "divide and conquer" approach (e.g., quicksort, mergesort, binary search). In math terms, the recursive part is

$$S(n) = cS(n/2) + g(n), \quad \text{for } n = 2^m, n > 1$$

- For problems that fit this case, the "expand, guess, verify" method produces the following:

$$S(n) = c^{\log n} S(1) + \sum_{i=\log n}^{n} (c^{\log n - i} g(2^i))$$

Slide 6

Examples

- Word problem: problem #18 in section 2.5 of textbook.
- Second-order: Fibonacci sequence (problems 24 in 2.4, 31 in 2.5).
- Divide-and-conquer: practice #25 in textbook. (Next time.)

Slide 7

Minute Essay

- None really — just sign in, unless you have questions.