Slide 1

Administrivia

- Reminder(?): Quiz 5 Tuesday. Likely topic is sets and counting (chapter 3 up to but not including probability).
- Homework(s) on chapter 3 coming soon.
- (Review minute essay from last time.)

Slide 2

Probability — Not-Equally-Likely Outcomes

- One approach — extend previous definition (size of "event" divided by size of sample space) by adding duplicates to sample space for outcomes that are more likely.
- Another approach — "probability distribution": For each x in sample space S, assign x a probability $p(x)$, such that

$$0 \leq p(x) \leq 1, \text{for all } x \in S$$

$$\sum_{x \in S} p(x) = 1$$

- Now for event E ($E \subseteq S$), we have

$$P(E) = \sum_{x \in E} p(x)$$

- Note that equally-likely-outcomes definition is a special case of the above.

Conditional Probability

Slide 3

- We could also consider, for two possibly related events E_1 and E_2, how likely it is that E_2 happens given that E_1 has happened — "conditional probability":

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

  Intuitive idea is that here the "sample space" is limited to E_1.

- If it turns out that $P(E_2|E_1) = P(E_2)$, we call E_1 and E_2 "independent events". In this case, we can derive that $P(E_1 \cap E_2)$ is — what?

- Notice resemblance between this and multiplication principle, and between rule for $P(E_1 \cup E_2)$ and addition principle.

Examples

Slide 4

- If a fair coin is tossed four times, what's the probability of getting four heads? What's the probability that the last toss is a head given that the first three are heads?

Slide 5

Expected Value

- You probably know about computing weighted averages — from classes in which your grade is computed as, e.g., 50% exams, 20% homework, etc.
- "Expected value" is a generalization of this: Given a sample space S, a "random variable" X (function from S to $\mathbb{R}$), and a probability distribution p, define expected value of X thus:

$$E(X) = \sum_{x \in S} X(x)p(x)$$

  Intuitive idea — "average" value, where the average is weighted by how likely the different values are.

Slide 6

Average-Case Analysis of Algorithms

- Previously we talked about estimating worst-case execution time of algorithms — amount of "work" as a function of input size.
- We could also talk about average-case amount of work, based on idea of expected value: Sample space is set of all possible inputs. For input x, $X(x)$ is the amount of work for x and $p(x)$ is the probability of x.

  Example — example 72 in textbook.

Binary (and other) Relations — Preview

Slide 7

- Idea of a binary relation is to express relationship between pairs of elements of a set. Some interesting special cases:
  - Partial orderings — useful in working out how we could put things "in order", e.g., a set of tasks with (some) ordering dependencies.
  - Functions (of 1 variable).
- Generalization — "n-ary relation", also with interesting special cases:
  - Functions of more than 1 variable.
  - Relational databases.

Minute Essay

Slide 8

- If a fair coin is tossed four times, what's the probability of getting two heads and two tails given that there's at least one head and at least one tail?

Minute Essay Answer

Slide 9

- Sample space has 16 elements (different outcomes of flipping coin four times).

  If E_1 includes outcomes with at least one head and at least one tail, $P(E_1)$ is 14/16, because E_1 is all of the sample space except the “all heads” and “all tails” outcomes.

  If E_2 includes outcomes with two heads and two tails, $P(E_2)$ is 6/16, because there are $C(4, 2) = 6$ ways to choose the two tosses that come up heads.

  $E_1 \cap E_2$ is just E_2.

  So from definition, $P(E_2|E_1) = (6/16)/(14/16)$, i.e., 6/14.