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Probability — Not-Equally-Likely Outcomes

- One approach extend previous definition (size of "event" divided by size of sample space) by adding duplicates to sample space for outcomes that are more likely.
- Another approach "probability distribution": For each x in sample space S, assign x a probability p(x), such that

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$$0 \leq p(x) \leq 1,$$
 for all $x \in S$
$$\sum_{x \in S} p(x) = 1$$

• Now for event E ($E \subseteq S$), we have

$$P(E) = \sum_{x \in E} p(x)$$

• Note that equally-likely-outcomes definition is a special case of the above.

Conditional Probability

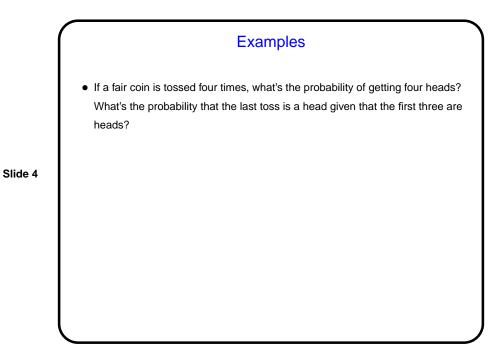
• We could also consider, for two possibly related events E_1 and E_2 , how likely it is that E_2 happens given that E_1 has happened — "conditional probability":

$$P(E_2|E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)}$$

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Intuitive idea is that here the "sample space" is limited to E_1 .

- If it turns out that $P(E_2|E_1) = P(E_2)$, we call E_1 and E_2 "independent events". In this case, we can derive that $P(E_1 \cap E_2)$ is what?
- Notice resemblance between this and multiplication principle, and between rule for $P(E_1\cup E_2)$ and addition principle.



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Expected Value

• You probably know about computing weighted averages — from classes in which your grade is computed as, e.g., 50% exams, 20% homework, etc.

• "Expected value" is a generalization of this: Given a sample space S, a "random variable" X (function from S to \mathbb{R}), and a probability distribution p, define expected value of X thus:

$$E(X) = \sum_{x \in S} X(x)p(x)$$

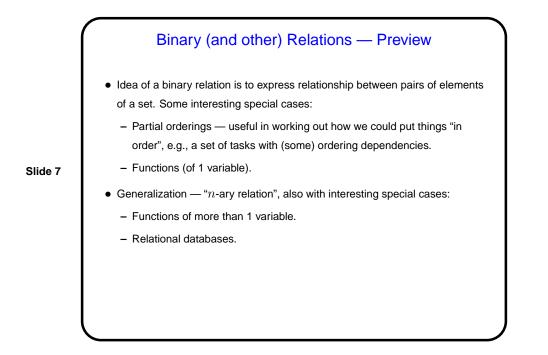
Intuitive idea — "average" value, where the average is weighted by how likely the different values are.

Average-Case Analysis of Algorithms

- Previously we talked about estimating worst-case execution time of algorithms — amount of "work" as a function of input size.
- We could also talk about average-case amount of work, based on idea of expected value: Sample space is set of all possible inputs. For input x, X(x) is the amount of work for x and p(x) is the probability of x. Example — example 72 in textbook.

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Minute Essay • If a fair coin is tossed four times, what's the probability of getting two heads and two tails given that there's at least one head and at least one tail? **Minute Essay Answer** • Sample space has 16 elements (different outcomes of flipping coin four times). If E_1 includes outcomes with at least one head and at least one tail, $P(E_1)$ is 14/16, because E_1 is all of the sample space except the "all heads" and "all tails" outcomes. If E_2 includes outcomes with two heads and two tails, $P(E_2)$ is 6/16, because there are C(4, 2) = 6 ways to choose the two tosses that come up heads. $E_1 \cap E_2$ is just E_2 . So from definition, $P(E_2|E_1) = (6/16)/(14/16)$, i.e., 6/14.

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