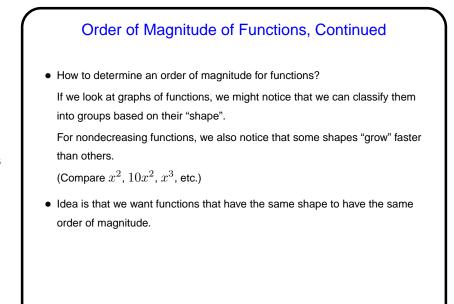


Slide 2

## Order of Magnitude of Functions By now you've probably heard "this is an O(N) algorithm", etc., many times. Here we'll define it formally. First: When we talked about analysis of algorithms (chapter 2), we came up with estimates of "total work" of the algorithm as a function of size of input ("problem size"). Useful and interesting, but a bit fine-grained — what we usually care about is behavior as problem size gets very big. So — idea is to come up with an "order of magnitude" for functions, analogous to "order of magnitude" for numbers. If the functions for two algorithms have the same order of magnitude, the functions are in some sense about equally fast/slow. Example: If you have two algorithms for processing an image with N pixels, one that takes time proportional to 1000N and one that takes time proportional to time N<sup>2</sup>, which do you pick? (Does the size of N matter?)



## Order of Magnitude of Functions, Continued

• Formal definition:

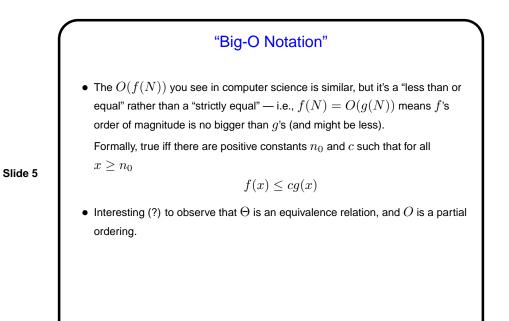
Write  $f=\Theta(g)$  to mean that f and g have the same order magnitude. Define to be true iff there are positive constants  $n_0,\,c_1,\,c_2$  such that for all  $x\ge n_0$ 

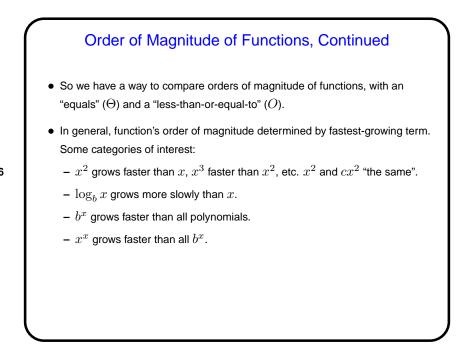
Slide 4

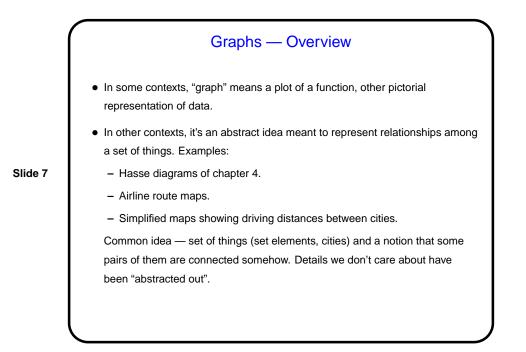
## $c_1 g(x) \le f(x) \le c_2 g(x)$

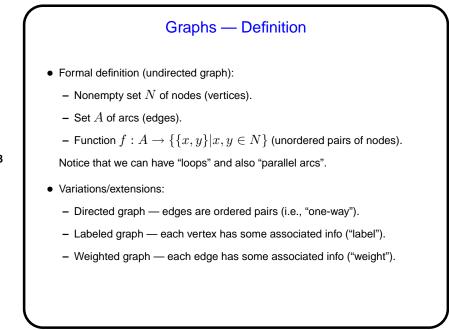
In other words, these functions are roughly proportional to each other.

• Can guess values  $c_1$ ,  $c_2$  and more or less show that they work by plotting resulting curves — but to really show that the definition holds, must do algebra to show. (Example.)



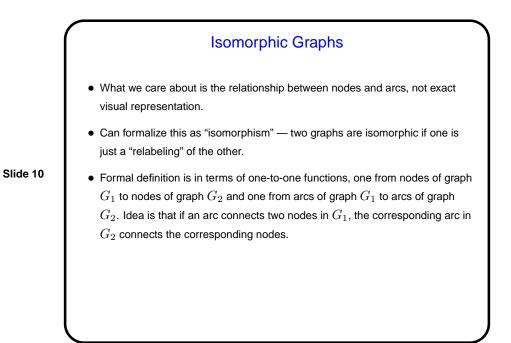


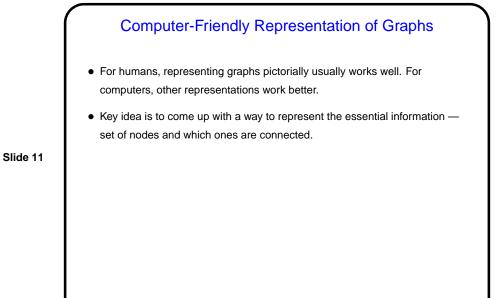


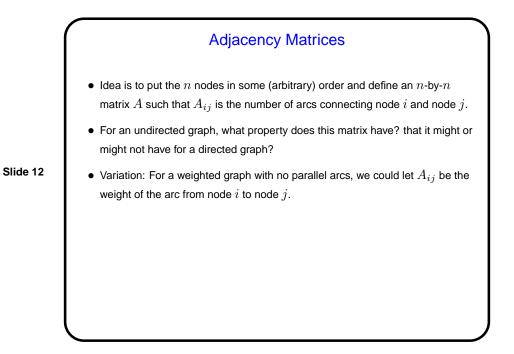


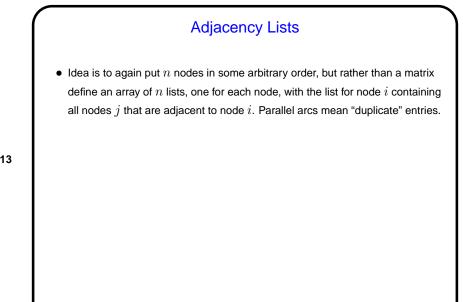


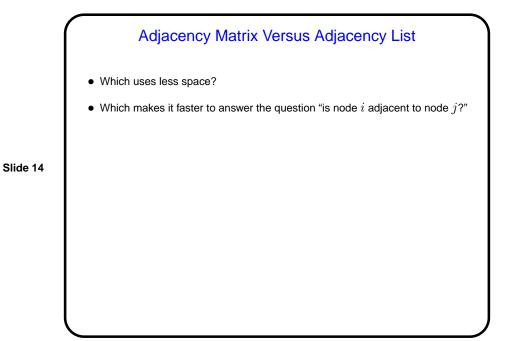
- Adjacent nodes (arc from one to the other).
- Loop, parallel arc.
- Simple graph (no loops or parallel arcs).
- Complete graph (every pair of nodes adjacent).
- Path (sequence of arcs), connected graph.
- Cycle, acyclic graph.

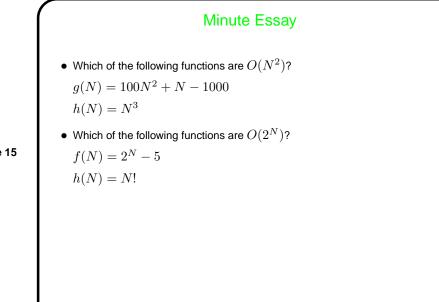












	Minute Essay Answer
16	• $O(N^2)$ ? $g(N) = 100N^2 + N - 1000 - yes$ $h(N) = N^3 - no$ • $O(2^N)$ ? $f(N) = 2^N - 5 - yes$ h(N) = N! - no