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First Principle of Mathematical Induction

- We can prove that P(k) is true for all integers $k \ge N$ (often N is 0 or 1, but not always) if we can show:
 - Base case: P(N)
 - Inductive step: For $k \ge N$, $P(k) \rightarrow P(k+1)$
 - That is: Assume P(k) and $k \ge N$ ("inductive hypothesis"), and show that then P(k+1)
- For readability/clarity, make this explicit, especially what you assume / have to show for inductive step.
- $\bullet\,$ Works because we have P(N) and then a chain of implications:

$$P(N) \rightarrow P(N+1), P(N+1) \rightarrow P(N+2), \dots$$

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First Principle of Mathematical Induction — Examples

• Example: Show that for $n \ge 1$,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

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Example: Show that for
$$n \ge 1$$
,

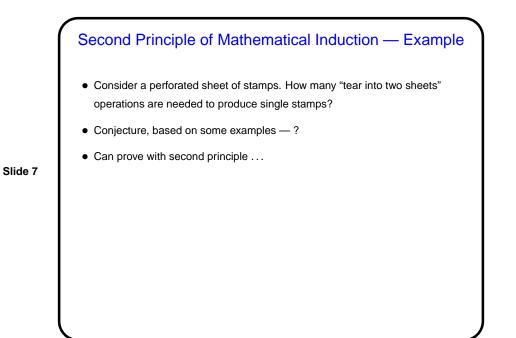
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Second Principle of Mathematical Induction

- Can also show that P(k) is true for all integers $k \ge N$ (often N is 0 or 1, but not always) if we can show that:
 - Base case: P(N)
 - Inductive step: For $k \geq N$, $((N \leq r \leq k) \rightarrow P(r)) \rightarrow P(k+1)$ That is: Assume that P(r) holds for all integers r with $N \leq r \leq k$, and that $k \geq N$ ("inductive hypothesis"), and show that then P(k+1)
- For readability/clarity, again make this explicit ...
- Notice inductive hypothesis here is more complicated, but gives you more to work with.
- $\bullet\,$ Works because we have P(N) and then a chain of implications:

$$P(N) \ \rightarrow \ P(N+1), P(N) \ \land \ P(N+1) \ \rightarrow \ P(N+2), \ldots$$

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Minute Essay • Is the material on induction making sense to you? Slide 8