Slide 1

Administrivia

- Reminder: Midterm next Tuesday. Review sheet on the Web.
- Reminder: Quiz 3 Thursday.
- Reminder: Homework 3 due today. Homework 4 officially due next week, but if you turn it in early you can pick up a sample solution before the exam.

Slide 2

Recursion and Recursive Definitions — Review/Recap

- Idea of recursion closely related to idea of induction — "build on previous smaller cases".
- First look at recursive definitions. To define something recursively:
  - Define one or more "base cases".
  - Define remaining cases in terms of other ("smaller") cases.
- Last time we looked at recursive definitions of sequences (define first (few) elements(s), others in terms of previous elements), sets (define base element(s), rules for generating other elements).

Slide 3

Recursive Definitions — Sets (Review)

- Example last time — integer arithmetic expressions. defined in terms of addition.
- More examples — strings, palindromes (from textbook).

Slide 4

Recursive Definitions — Operations (Review)

- Examples last time — factorial, multiplication of non-negative integers, defined in terms of addition.
- Example — (integer) division of a non-negative integer by a positive integer, defined in terms of subtraction.

Slide 5

Recursive Algorithms

- Recursive definitions of sequences or operations often can be turned into recursive algorithms with little effort.
- Simple classic example — function to compute n factorial.
- Efficiency considerations:
  - In terms of computer time/memory usage, recursion is almost always worse than iteration — but not always, and sometimes not much worse.
  - In terms of human effort to get program running correctly, recursion may be much better.
- Examples in text — selection sort and binary search. Quicksort and mergesort are other good ones.

Slide 6

Recursive Algorithms — Examples

- Examples in text — selection sort and binary search. Quicksort and mergesort are other good ones.
- Other good examples involve “trees” — not discussed yet, but a hierarchical file system (with folders and files) is an example, and we could sketch an algorithm to search one.

Slide 7

Reasoning About Recursive Algorithms

- A recursive algorithm "works" if:
  - It works for the base case(s).
  - For other cases, it works *assuming* the recursive calls work.
  - The recursion eventually stops — recursive calls are always "smaller", and eventually reduce to base cases.
- We could formalize this as a proof by induction.

Slide 8

Minute Essay

- Consider the following recursive definition of a sequence:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= 10S(n-1)+1, \quad \text{for } n > 1 \end{aligned}$$

  What are $S(1), S(2), \ldots S(5)$?
- We could use part or all of Thursday's class to review for the midterm. Would you be interested in doing this, and if so are there particular topics?

Slide 9

Minute Essay Answer

- The first few terms:

  $S(1) = 1$

  $S(2) = 11$

  $S(3) = 111$

  $S(4) = 1111$

  $S(5) = 11111$