Slide 1

Administrivia

- (None.)

Slide 2

Recurrence Relations

- Recall the silly example of defining a sequence recursively:

$$\begin{aligned} S(1) &= 1 \\ S(n) &= S(n-1) \times 10, \quad \text{for } n > 1 \end{aligned}$$

  Expanding out some terms, it seems fairly obvious that an equivalent definition would be $S(n) = 10^{n-1}$ — a "closed-form solution" to the recurrence relation given in the second line of the definition.

- We'll look at various ways to get from a recursive definition to a closed-form one, because the latter are easier to compute, but sometimes it will be much easier to write down the definition recursively.

Slide 3

Solving Recurrence Relations, Continued

- For the silly example

$$
\begin{aligned}
S(1) &= 1 \\
S(n) &= S(n-1) \times 10, \quad \text{for } n > 1
\end{aligned}
$$

  we guessed a solution of $S(n) = 10^{n-1}$. Can we verify that this is the same as the recursive definition? yes, via a proof by induction ...

- Call this method "expand, guess, verify".
- Try another example — section 2.5 problem 3.

Slide 4

Solving Recurrence Relations, Continued

- Is there another way? In general, probably not, but there are some formulas for some frequently-occurring special cases.
- One is "first-order linear" recurrence relations with constant coefficients. If

$$S(n) = cS(n-1) + g(n)$$

  then we can show (see textbook for derivation) that

$$S(n) = c^{n-1}S(1) + \sum_{i=2}^{n}(c^{n-i}g(i))$$

- Apply this to the two problems we did earlier — we should get the same results.

Slide 5

Another Special Case

- Another case for which there's a formula — two base cases, and the recursive part of the definition depends on the previous two elements ("second-order") in a simple way:

$$S(n) = c_1 S(n-1) + c_2 S(n-2)$$

- For problems that fit this case, solution is more complicated (the "guess" part of getting it seems to involve a rabbit and a hat, though verifying it is relatively straightforward).

Slide 6

Another Special Case — Solution

- First find roots r_1 and r_2 of

$$t^2 - c_1 t - c_2 = 0$$

- and then (if $r_1 \neq r_2$) find p and q to solve

$$p + q = S(1)$$

$$pr_1 + qr_2 = S(2)$$

- and then the formula is

$$S(n) = pr_1^{n-1} + qr_2^{n-1}$$

- Example: Fibonacci sequence (problems 24 in 2.4, 31 in 2.5).

Minute Essay

- None — sign in.

Slide 7