Administrivia

Slide 1

- Next homework will be on the Web later today / early tomorrow. (I will send e-mail.) Due next Thursday.

Permutations

Slide 2

- We might want to know how many ways we can choose an ordered sequence of r objects, chosen from n possibilities with no repeats. Call this $P(n, r)$.

  Example: How many 7-digit phone numbers have no repeated digits?
- Can we come up with a general formula? (Of course. Let's derive one.)
- Look at some boundary cases — $r = n$, $r = 0$, $r = 1$, etc. (We'll need to agree that $0! = 1$.)

Slide 3

Combinations

- Or we might want to know how many ways we can choose an *unordered* collection of r objects, chosen from n possibilities with no repeats. Call this $C(n, r)$.

  Example: How many ways can we draw 5 cards from a deck of 52?
- Can we come up with a general formula? (Of course. Let's derive one.)
- Again look at some boundary cases — $r = n$, $r = 1$, $r = 0$.
- (Another common notation for this is $\binom{n}{r}$ ("n choose r").)

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Permutations Versus Combinations

- In general: If order matters, it's a permutation; if order doesn't matter, it's a combination.
- (Contrast "how many phone numbers with no repeated digits" (order matters) with "how many 5-card hands?" (order doesn't matter).)

Potential Pitfall — Counting Things Twice

- A problem is that some proposed solutions sound reasonable but actually manage to count some things twice, or don't count some things at all.
- Example: example 55 part (d). (Not in class, but you should review.)

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Permutations and Combinations With Repetitions

- Definitions of $P(n, r)$ and $C(n, r)$ specified "without repeats". What if we want to allow repeats?
- For permutations, not too tough — n^r ways to choose an ordered sequence of r things from n possibilities, if we allow repeats?
- For combinations, it's trickier. How many ways can we choose an unordered collection of r things from n possibilities, if we allow repeats? Use a clever idea from example 58.

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Permutations and Combinations, Example(s)

- How many ways to draw 5 cards from a standard 52-card deck and get three of a kind and a pair?

Slide 7

Probability — Equally-Likely Outcomes

- Basic definition: If S ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and E ("event") is a subset of S, then we define the probability of E as

$$P(E) = \frac{|E|}{|S|}$$

  Examples: Sequences of coin tosses, 5-card "hands" chosen from 52-card deck, etc.

- Note that $0 \leq P(E) \leq 1$. (Why?) When is $P(E) = 0$? When is $P(E) = 1$?

- Note that we can apply anything we know about sizes of sets. (E.g., if E_1 and E_2 are disjoint, what is $P(E_1 \cup E_2)$ in terms of $P(E_1)$ and $P(E_2)$?)

Slide 8

Slide 9

Example(s)

- In a group of n people, what's the probability that at least two people have the same birthday?