CSCI 1323 (Discrete Structures), Spring 2013
Derivation Rules for Propositional Logic

Equivalence rules

Expression	Equivalent to	Name/abbreviation
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative — comm
$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	Associative — ass
$(P \vee Q)'$ $(P \wedge Q)'$	$P' \wedge Q'$ $P' \vee Q'$	De Morgan's Laws — De Morgan
$P \to Q$	$P' \vee Q$	Implication — imp
P	$(P')'$	Double negation — dn

Inference rules

From	Can derive	Name/abbreviation
P, $P \to Q$	Q	Modus ponens — mp
$P \to Q$, Q'	P'	Modus tollens — mt
P, Q	$P \wedge Q$	Conjunction — con
$P \wedge Q$	P, Q	Simplification — sim
P	$P \vee Q$	Addition — add

Deduction method

To prove

$$(P_1 \wedge P_2 \wedge \ldots \wedge P_n) \to (R \to S)$$

it is enough to prove that

$$(P_1 \wedge P_2 \wedge \ldots \wedge P_n \wedge R) \to S$$

More inference rules

From	Can derive	Name/abbreviation
$P \to Q$, $Q \to R$	$P \to R$	Hypothetical syllogism — hs [Example 16 in 1.2]
$P \vee Q$, P'	Q	Disjunctive syllogism — ds [Exercise 23 in 1.2]
$P \to Q$	$Q' \to P'$	Contraposition — cont [Exercise 24 in 1.2]
$Q' \to P'$	$P \to Q$	Contraposition — cont [Exercise 25 in 1.2]
P	$P \wedge P$	Self-reference — self [Exercise 26 in 1.2]
$P \vee P$	P	Self-reference — self [Exercise 27 in 1.2]
$(P \wedge Q) \to R$	$P \to (Q \to R)$	Exportation — exp [Exercise 30 in 1.2]
P, P'	Q	Inconsistency — inc [Exercise 31 in 1.2]
$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$	Distributive — dist [Exercise 19 in 1.2]
$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$	Distributive — dist [Exercise 20 in 1.2]