Administrivia

Slide 1

- Reminder: Homework 2 due today.
- If you missed the quiz last week, I don't normally allow make-ups (but I do drop the lowest score). If you want to test yourself, though, ask me for a copy before looking at the (posted) solution.
- Review sheet for midterm on the Web; also sample solutions for quizzes. Sample solutions for homeworks coming, in hardcopy.

Representing Data

Slide 2

- To the hardware "it's all ones and zeros". But those ones and zeros can encode numbers (various forms), text, etc.
- Numbers in particular are interesting because we want to implement arithmetic operations.
- In theory you learned about integer representation and arithmetic in CSCI 1320. Review . . .

Binary Versus Decimal (Review?)

- In decimal (base 10) notation, each digit is multiplied by a power of 10. Same idea for binary (base 2), but using powers of 2.
- So, converting from binary to decimal is easy (if tedious), working from definition. Example?

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Binary Versus Decimal, Continued

- Converting from decimal to binary? Repeatedly divide by 2 and record remainders . . .

  We could describe this as a recursive algorithm for computing $bits(n)$:

  - Base case is $n < 2$; trivial.
  - For recursive step, divide n by 2 to get quotient q and remainder r. Then $n = 2q + r$, and:

    The last bit of $bits(n)$ should be r.

    The remaining bits are $bits(q)$, left-shifted by 1.

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Binary Versus Decimal, Continued

Slide 5

- Terminology: "Least significant" and "most significant" bits.
- Seems like there would be one obvious way to store the multiple bytes of one of these in memory, but no — "big endian" versus "little endian" (names based on *Gulliver's Travels*).

Binary Versus Decimal, Continued

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- Binary is useful for showing real internal state but not very compact. Decimal is compact but not so easy to convert to/from binary.
- We might notice — easy to convert to/from a base that's a power of 2. Hence the use of "octal" (base 8) and "hexadecimal" (base 16). For the latter, we need more than 10 digits, so we use "A" through "F".

  Examples?
- Notice that we can also convert directly to/from decimal, much as we did for binary.

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Representing Integers (Review?)

- Representing non-negative integers is easy — convert to binary and pad on the left with zeros.
- What about negative integers?
- Could try using one bit for sign, but then you have +0 and -0, and there are other complications.
- Or . . . consider a car odometer — in effect, representable numbers form a circle, since adding 1 to largest number yields 0.

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Representing Integers, Continued

- We could implement the car-odometer idea in binary, and then choose where to "cut the circle" (between smallest and largest):
  - Between 0 and all ones — unsigned integers.
  - Between largest number with 0 as the MSB and smallest number with 1 as MSB — "two's complement" signed integers.
- Notice that with the two's complement scheme, +1/-1 moves us "around the circle" — nothing special needed for negative numbers.
- Notice that if we have n bits, adding 2^n to x gives us x again. This leads to an easy way to compute $-x$: Compute $2^n - x$, and notice that

  $$2^n - x \ = \ (2^n - 1) - x + 1$$

  which is very easy to compute . . .

  Examples?

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Signed Versus Unsigned

- If we have n bits, we can use them to represent signed values in — what range?

  Or we can use them to represent non-negative values only ("unsigned values") — then what range?

- Many MIPS instructions have "unsigned" counterparts — `addu`, `addiu`, `sltu`, etc.

- Example: Suppose we have

  0x00000000 in `$t0`

  0xfffffff2 in `$t1`

  What happens if we execute `slt $t2, $t0, $t1`?

  What happens if we execute `sltu $t2, $t0, $t1`?

  (Same bits, different interpretations!)

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Sign Extension

- If we have a number in 16-bit two's complement notation (e.g., the constant in an I-format instruction), do we know how to "extend" it into a 32-bit number?

  For non-negative numbers, easy.

  For negative numbers, also not too hard — consider taking absolute value, extending it, then taking negative again.

- In effect — "extend" by duplicating sign bit.

- (Notice that not all instructions that include a 16-bit constant do this.)

Slide 11

Two's Complement and Addition/Subtraction

- Addition in binary works much like addition in decimal (taking into account the different bases). Notice what happens if one number is negative. (Try an example or two.)
- Subtraction could also be done the way we do in decimal. Or how else could we do it? (Again, try some examples.)
- But there is one catch, related to the fact that operands and result are all n bits. What is it?

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Addition/Subtraction and Overflow

- If adding two n-bit numbers, result can be too big to fit in n bits — "overflow".
- For unsigned numbers, how could we tell this had happened?
- How about for signed numbers?

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Addition/Subtraction and Overflow, Continued

- Notice that we can't get overflow unless input operands have the same sign.
- If we add two positive numbers and get overflow, how can we tell this has happened? Does this always work?
- If we add two negative numbers and get overflow, how can we tell this has happened? Does this always work?

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Addition/Subtraction and Overflow, Continued

- When we detect overflow, what do we do about it?
- From a HLL standpoint, we could ignore it, crash the program, set a flag, etc.
- To support various HLL choices, MIPS architecture includes two kinds of addition instructions:
  - Unsigned addition just ignores overflow.
  - Signed addition detects overflow and "generates an exception" (interrupt) — hardware branches to a fixed address ("exception handler"), usually containing operating system code to take appropriate action.

  This is why, if you look at MIPS assembler for C programs, the arithmetic is unsigned — C ignores overflow, so why bother to look for it.

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Representing Real (Non-Integer) Numbers

- Approach is based on a binary version of “scientific notation”:
  In base 10, we can write numbers in the form $+/-x.yyyy \times 10^z$.
  E.g., $428 = 4.28 \times 10^2$, or $-.0012 = -1.2 \times 10^{-3}$.
- We can do the same thing in base 2. Examples:
  $32 = 1.0_2 \times 2^5$
  $-3 = -1.1_2 \times 2^1$
  $1/2 = 1.0_2 \times 2^{-1}$
  $3/8 = 1.1_2 \times 2^{-2}$
- This is “floating point” (as opposed to “fixed point”, which would allow for non-integers but wouldn’t allow as much flexibility — wide range, all with reasonable precision).

Slide 16

Representing Real Numbers, Continued

- In base 10, we can completely specify a number by giving its sign, a number in the range $0 \leq x < 10$ (the “significand” or “mantissa”), and the exponent for 10. Same idea applies in base 2.
- So, most/all “floating-point formats” have a bit for the sign, some bits for the significand, and some bits for the exponent. Different choices are possible, even with the same total number of bits; (at least) one architecture (VAX) even supported more than one format with the same number of bits(!).
- With integers, number of bits limits the range of numbers that can be represented. With “floating-point” numbers, two limiting factors — number of bits for the significand (which limits what?), and number of bits for the exponent (which limits what?).
  (Does this suggest why the VAX designers offered two formats?)
  (To be continued . . .)

Minute Essay

- Was anything today *not* review?
- Anything you're particularly interested in reviewing next time?

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