Sustainable Withdrawal and Accumulation Rates*

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Abstract

Determining a sustainable withdrawal rate is an important factor to consider in retirement investment planning. An equally important problem is determining an accumulation rate yielding an investment large enough for retirement. Both these problems involve predicting the effect of periodic incremental changes to an investment.

Making predictions using historical averages does not yield accurate solutions because of the large variability in security returns. Instead we use use a different measure: Given an initial investment and a periodic increment, what is the probability of reaching the desired investment target given a particular investment strategy? We illustrate the utility of this approach by considering

- the accumulation rates yielding a desired retirement goal,
- the tradeoff between placing retirement savings in taxed and tax-deferred accounts,
- the effect on retirement income of decreasing the the investment's stock portion annually,
- the importance of considering different assets in investment planning,
- the size of an investment resulting from annually saving in an individual retirement account,
- the accumulation rate sufficient to pay for college education, and
- the risk-reward tradeoffs of various assets on short-term investments.

Most importantly, we empirically explore the robustness of our analyses by considering the effect of small decreases in return rates and small variations in investment time periods. Finally, we precisely describe our algorithm, illustrating that these investment calculations are not computationally difficult.

1 The Incremental Investment Problem

The *incremental investment problem* generalizes the retirement withdrawal problem and the associated accumulation problem. Given

- an initial investment value v_0,
- an amount I_t to add to the investment on a periodic basis,

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- the number of investment periods n, and
- an investment strategy,

the problem is to determine the final investment value. As we will soon argue, using historical data causes our predictions to vary greatly so we will instead additionally require

- a desired final investment value v_d,

and compute the probability that the final investment value meets or exceeds the desired final investment value, i.e., $v_n \geq v_d$. For example, a young person may save $2000 annually for forty years to pay for retirement. If the goal is to save $400,000, the goal is attained if the final investment value is greater than or equal to $400,000. A retired person may have a $1,000,000 investment and wish to increment the investment by $-$30,000 per year for thirty years. The goal might be to end with the same investment value of $1,000,000.

Many different variants of the incremental investment problem exist. In the general problem, the amount I_t to increment varies with time, but, in this paper, we consider only time-independent increments I or inflated increments. In the *inflated increment investment problem*, the amount I to invest regularly is inflated according to the U.S. Bureau of Labor Statistics's Consumer Price Index [BoLS] or some other inflation indices. Inflating the final investment value but not necessarily the increment I yields the *inflated-target increment investment problem*. Problems vary according to the *time frame*, i.e., the total time duration, and according to the *increment time period*, i.e., the time between increments. In the young person example above, the time frame is forty years with an increment time period of one year.

Incremental investment problems also differ according to asset allocations. A *fixed asset allocation* apportioning fixed investment proportions to different asset classes contrasts with a *varying asset allocation* permitting different classes' investment proportions to vary arbitrarily or according to a fixed formula. For example, a formula may specify that the percentage allocated to stocks is the maximum of 100% and $125 -$ *age*, where *age* represents a person's age in years. Asset allocations also differ according to the assets under consideration.

The taxation of the investment also affects the problem. Retirement savings usually occur in tax-deferred accounts with taxes paid at income tax rates upon withdrawal. Investments in taxed accounts incur taxation as dividends are earned and when capital gains are realized. For example, stock dividends are taxed in the year, but appreciation in the stock's value is taxed only in the year when sold. In this paper, we will ignore taxes paid when withdrawing money from tax-deferred accounts, assuming that taxes will be paid from the withdrawn amount.

1.1 Withdrawal and Accumulation Problems Differ

The incremental investment problem encompasses both the withdrawal problem and the associated accumulation problem raising the question how the withdrawal and accumulation problems differ. Intuitively, a withdrawal problem may begin with an investment, incrementing a negative amount every period and ending with no investment. An accumulation problem "runs times backwards" starting with no investment, incrementing by a positive amount every period and ending with an investment. Thus, one might be tempted to use a table of accumulation data to solve a withdrawal problem by just negating the increment and declare there is no need to separately investigate the two problems.

Given the same market conditions, the accumulation and withdrawal problems react differently. The accumulation problem yields the largest final value when adding money during down market periods, while the withdrawal problem ends with the largest value during up market periods. Intuitively, each dollar added to an investment purchases more shares when share prices are low, while each dollar withdrawn requires fewer shares when share prices are high. More mathematically, consider a simulation with three time periods 0, 1, and 2. Assume share prices p and p are fixed for both problems, initially n shares are owned, and that I is invested at time 1. Thus, the investment's final value is $p(n + I/p)$. Assuming p and n are fixed, this value is maximized for a positive I by minimizing p and

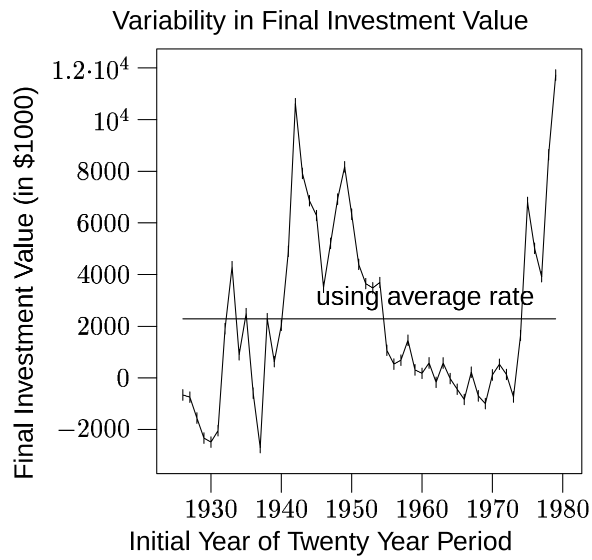

Figure 1: Final investment values of twenty-year timeframes for $1.0 million invested in large-company stocks withdrawing $90,000 annually.

for a negative I by maximizing p. Thus the final ending value for the accumulation and withdrawal problems differ under the same market conditions.

1.2 Measuring Success Using Probabilities

For the incremental investment problem, using average historical return rates yields misleading results because of the large standard deviation in the return rates. Instead we use exact historical return rates, computing the probability of reaching the desired financial target.

Using average historical return rates $\overline{r}$, the incremental investment problem has succinct mathematical representations. As a recurrence equation,

$$v_t = \begin{cases} v_0 & \text{if } t = 0 \\ v_{t-1}(1+\overline{r}) + I & \text{otherwise} \end{cases}$$

At each time period, the investment's value is increased by the average rate and its value is incremented by I. As a closed formula,

$$v_t = v_0(1+\overline{r})^t + I\frac{(1+\overline{r})^t - 1}{\overline{r}}.$$

Basically, the value grows exponentially with time. Unfortunately, the large standard deviation in historical return rates reduces the usefulness of this approximation. The standard deviation for large-company stocks over 1926–1997 was 20.3% with an annual geometric mean of 11.0% [Ass99, Chapter 6]. Withdrawing $90,000 annually for twenty years from $1,000,000 invested totally in large-company stocks will yield an investment worth $2.28 million according to the formula above. Figure 1 shows the final investment value for the fifty-four twenty-year periods starting in years 1926–1979. Although the mean is about the same, the values range from a high of $11.7 million to $-2.7 million, with fifteen time periods resulting in bankruptcies. This clearly shows that using average historic return rates does not yield useful predictions.

claim that using averages even when using historical data is not useful

2 Effect of Asset Choice

claims:

- small-cap can be significantly better than large-cap
- intermediate-term govt bonds outperform long-term government bonds
- lt corp $\geq$ lt govt
- not necessarily transitive, e.g., ltcb vs. itgb; treas can outperform ltgb in some cases
- JDO: what question(s) asking
- Craig: what experiments we performed justification for table differences
- JDO: describe our results
- interpret our results
  - Craig: bonds
  - JDO: stocks

charts: lcoVltcb.txt vs. scoVltcb.txt lcoVltcb.txt vs. scoVltgb.txt lcoVltgb.txt vs. scoVitgb.txt

For the retirement withdrawal problem, Cooley et al. *Finish: Add citation* considered only large-company stocks and long-term corporate bonds (with one additional table incorporating U.S. Treasury bills). We investigate the effect of other asset classes on the probability of success. To avoid an exponential number of comparisons, we restrict our investigation to pairwise comparisons of assets.

Craig, we need to mention use of small-company stocks and long-term corporate bonds, which are used only for this experiment, not for others. tax-free, fixed asset allocation, comparisons close to Cooley et al., use of table subtractions

Tables 1, 2, and 3 sample the results. Significant differences are mainly found along the upper-sloping diagonal because the left columns, representing small withdrawals, usually have high success probabilities particularly for higher stock allocations and the right columns, representing large withdrawals, usually have low success probabilities particularly for lower stock allocations. As expected, the bottom rows of Table 1 contain only zeroes because these represent the same bond-only allocations in both simulations. Similarly, the top rows in Tables 2 and 3 contain only zeroes.*Finish: this paragraph and more.*

References

[Ass99] Ibbotson Associates. *Stocks, Bonds, Bills, and Inflation 1999 Yearbook.* Ibbotson Associates, Chicago, 1999.

[BoLS] U.S. Bureau of Labor Statistics. Consumer prices indexes. http://stats.bls.gov/cpihome.htm.

asset allocation	payout period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100/0	20 years	0.2	2.4	-2.1	-11.9	-15.8	-22.2	-30.5	-32.2	-30.7	-23.7
	25 years	1.0	1.9	-10.0	-20.1	-23.4	-30.4	-31.5	-26.5	-19.6	-14.2
	30 years	1.0	0.2	-10.1	-19.0	-24.4	-33.7	-32.3	-24.6	-23.4	-18.9
75/25	20 years	0.0	0.0	-6.9	-15.9	-20.3	-29.5	-33.6	-29.3	-22.3	-17.3
	25 years	0.0	0.0	-16.1	-28.8	-33.3	-31.6	-25.2	-20.8	-14.2	-9.7
	30 years	0.0	-4.4	-16.9	-28.0	-34.8	-31.7	-28.7	-22.1	-15.3	-10.8
50/50	20 years	0.0	0.0	-9.6	-19.6	-30.5	-33.3	-23.7	-14.3	-8.3	-4.9
	25 years	0.0	-2.1	-21.0	-38.0	-32.1	-23.1	-11.3	-7.1	-4.4	-3.5
	30 years	0.0	-10.6	-29.8	-40.4	-35.2	-17.4	-11.2	-6.0	-4.1	-3.3
25/75	20 years	0.0	0.0	-13.0	-27.7	-15.7	-6.0	-7.6	-5.3	-3.1	-1.4
	25 years	0.0	-9.5	-39.0	-12.6	-4.9	-9.7	-5.9	-2.1	-1.0	-0.3
	30 years	0.0	-33.3	-23.6	-4.6	-12.0	-8.9	-1.9	-0.8	0.0	0.0
0/100	20 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	25 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 1: Difference of success probability between large-company and small-company stocks when paired with long-term corporate bonds. Asset allocations indicate the ratio of stocks to bonds. A positive (negative) number indicates a higher success for large- (small-) company stocks.

asset allocation	payout period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100/0	20 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	25 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75/25	20 years	0.0	0.0	0.8	1.2	0.9	1.9	0.5	0.7	0.5	0.9
	25 years	0.0	0.0	2.1	0.3	1.2	1.2	0.5	1.2	0.3	0.3
	30 years	0.0	2.4	1.7	-0.2	0.8	1.5	1.7	0.7	0.2	0.0
50/50	20 years	0.0	0.0	1.9	-0.2	4.4	3.5	5.0	2.2	0.8	0.0
	25 years	0.0	2.8	4.1	2.2	3.5	2.9	1.9	0.2	0.3	0.0
	30 years	0.0	3.7	2.3	5.0	2.5	2.5	0.4	0.4	0.0	0.0
25/75	20 years	0.0	0.2	4.0	5.3	6.5	6.0	3.4	2.7	0.0	0.0
	25 years	0.0	6.4	3.1	3.3	5.9	2.4	0.7	0.0	0.0	0.0
	30 years	0.0	10.0	5.2	7.9	2.7	0.4	0.0	0.0	0.0	0.0
0/100	20 years	0.0	6.7	12.2	4.1	1.7	2.3	8.3	0.0	0.0	0.0
	25 years	1.7	12.3	1.9	2.5	7.5	0.9	0.0	0.0	0.0	0.0
	30 years	31.7	1.9	2.5	9.3	0.0	0.0	0.0	0.0	0.0	0.0

Table 2: Difference of success probability between long-term corporate bonds and long-term government bonds when paired with large-company stocks. Asset allocations indicate the ratio of stocks to bonds. A positive (negative) number indicates a higher success for long-term corporate (government) bonds.

asset allocation	payout period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100/0	20 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	25 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	30 years	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
75/25	20 years	0.0	0.0	-4.8	-1.1	0.2	-2.7	-0.8	-0.2	-0.3	-0.9
	25 years	0.0	0.0	-4.3	-1.9	0.0	1.0	-0.2	-2.6	-0.1	-0.3
	30 years	0.0	-6.6	-1.7	0.8	0.4	0.0	-2.1	-0.9	-0.2	0.0
50/50	20 years	0.0	0.0	-10.7	-0.4	-4.7	-0.8	-6.0	0.5	0.0	0.3
	25 years	0.0	-4.9	-8.4	0.0	-0.4	-2.0	0.5	0.0	0.0	0.0
	30 years	0.0	-14.3	-6.9	-0.1	-2.5	1.3	0.0	0.0	0.0	0.0
25/75	20 years	0.0	-0.2	-13.7	-9.2	-10.1	0.7	0.5	0.0	0.0	0.0
	25 years	0.0	-15.9	-12.1	-6.2	1.5	1.3	0.0	0.0	0.0	0.0
	30 years	0.0	-29.9	-13.9	2.4	2.1	0.0	0.0	0.0	0.0	0.0
0/100	20 years	0.0	-7.3	-30.3	-6.6	0.8	3.0	0.0	0.0	0.0	0.0
	25 years	3.0	-40.2	1.2	1.4	1.9	0.0	0.0	0.0	0.0	0.0
	30 years	-27.1	-3.7	2.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0

Table 3: Difference of success probability between long-term government and intermediate-term government bonds when paired with large-company stocks. Asset allocations indicate the ratio of stocks to bonds. A positive (negative) number indicates a higher success for long- (intermediate-) term government bonds.