Maximum Flow

11-20-2003

Opening Discussion

- Last time we introduced an all-pairs shortest path algorithm. Who can remind me of how that algorithm worked? What was the order of it?
- What makes TSP different from general shortest-path?
- Do you have any questions about the assignment?

Floyd-Warshall All-Pairs Shortest Path

- This algorithm is $O(V^3)$. To do this we consider a subset of the vertices in the graph with n vertices that only has the first k. Given any pair of vertices in k, i an j, there are two possibilities for the shortest path between them with intermediates in the first k vertices.
  - If k is not in the path then the shortest path with the subset 1..k-1 is the same as the path with the subset 1..k.
  - Otherwise there is a path from i to k, p1, and a path from k to j, p2. Those paths don't involve

More Floyd-Warshall

- Given this we can made a different recurrence relationship.

$$d_{ij}^{(k)} = \begin{cases} w_{ij} & k = 0 \\ \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & k \geq 1 \end{cases}$$

- We want to do this bottom up though so we work through the matrix applying this rule to each ij pair and increment k until all vertices are included.

Johnson's Algorithm for Sparse Graphs

- There is another algorithm for finding all pair shortest paths that is efficient for sparse graphs. This algorithm uses both Bellman-Ford and Dijkstra's algorithms.
- We aren't going to cover the details in class, but you should know that when done well it can solve the problem in O(VE) time.

The Maximum Flow Problem

- In this problem we are looking at the rate at which something moves through a system. The graph formalism gives us a way to represent it independent of the underlying problem.
- Examples could be water flowing through pipes or bottle of Coke moving through the bottling plant.
- The weighting on edges is a maximum flow rate. Material is not allowed to build up at the vertices so input must equal output except at sources and sinks.

Flow Networks

- In a flow network the weight of edges is called their capacity. The network has two special vertices: a source s and a sink t.
- A flow is a real-valued function f: V×V→R such that:
  - For all u,v in V, f(u,v)≤c(u,v).
  - For all u,v in V, f(u,v)=-f(v,u).
  - For all u in V-{s,t}, the sum over all v in V of f(u,v)=0.

Minute Essay

- I'm trying to decide what algorithm to have you do for assignment #6. It will be a graph algorithm that we have discussed, but I want it to tell you something interesting about the site itself. Do you have any suggestions?
- Test code for assignment #5 is due today.