Difference Equations and Chaos

10-3-2005

Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?

Population Modeling

- Last time we did some differential equations for physics. This time we want to look at a slightly different system: population biology.
- It turns out these types of equations also work for many chemical systems as well.
- The basic equations are the Lotka-Volterra System. N is prey, P is predators, a, b, c, and d are positive constants. This is over-simplistic, but suites our needs well.

$$\frac{dN}{dt} = N(a - bP)$$

$$\frac{dP}{dt} = p(cN - d)$$

Difference Equations

- Similar to differential equations are difference equations. These are discrete equations where we calculate the next value of the system from the previous one.
- These systems are sometimes referred to as mappings.
- Our numerical solutions to differential equations actually convert them to difference equations.

The Logistic Map

- A common example of a map is the logistic map. The formula for the logistic map is extremely simple: $x_{n+1}=rx_n(1-x_n)$.
- Iterating a simple 1-D map like this can be viewed by drawing a “cobweb diagram”. The formula is quadratic in x, but opens downward.
- Fixed points are places where the curve crosses the y=x line. Depending on the slope at that the intersection the fixed points might be stable or unstable.
- Let's see if we can write code to draw a cobweb diagram.

Bifurcation

- If we vary the value of r, interesting things happen to the behavior of this system. At small values there is a single, stable fixed point. At larger values, that fixed point becomes unstable and we get a period-2 cycle instead. This split is called a bifurcation.
- Increasing the value a bit more produces another bifurcation to a period-4 cycle.
- The separation between bifurcations gets smaller and the system actually becomes chaotic.
- Let's write code to draw a bifurcation diagram.

Reminders

- Remember to turn in assignment #5 by midnight tonight.