Differential Equations

9-29-2006

Opening Discussion

- What did we talk about last class?
- Let's talk a bit about the first project.
- What do you know about differential equations?

What are ODEs?

- Ordinary differential equations are extremely common in science. The idea is that we have functions that tell us the derivatives of values instead of telling us the values themselves.
- In a math class you learn various techniques to solve ODEs. The simplest ODEs can be solved simply by integrating them.
- Numerically, we can approximate the function by “following the slope” since it is the slope that we are given.

$$\dot{x}_1 = f_1(t, x_{1,} x_{2,..}, x_n)$$
$$\dot{x}_2 = f_2(t, x_{1,} x_{2,..}, x_n)$$
$$\ldots$$
$$\dot{x}_n = f_n(t, x_{1,} x_{2,..}, x_n)$$

ODEs in Matlab

- Matlab has a number of build in ODE solvers. All of these deal with systems of linear ODEs. That means that we have a set of equations of the form where the first derivative of a value is equal to some function.
- In general, any ODE of any order (those involving higher derivatives) can be converted to a system of first order ODEs by using variables to represent the higher order derivatives.
- Unless you have a reason to use something else, you will typically solve your ODEs with the ode45 function. This function uses a 4^{th} to 5^{th} order Runga-Kutta method.

Euler's Method

- Just to help you see how we solve differential equations on a computer, we should look at Euler's method. This is a first order method that you shouldn't use unless you have nothing better to use. It has the advantage of being simple and fast.
- Given the system of equations mentioned before, Euler's method would say the x values are as follows.

$$x_1(t+\Delta t)=x_1(t)+\Delta t*f_1(t, x_{1,} x_{2,}.., x_n)$$
$$x_2(t+\Delta t)=x_2(t)+\Delta t*f_2(t, x_{1,} x_{2,}.., x_n)$$
$$...$$
$$x_n(t+\Delta t)=x_n(t)+\Delta t*f_n(t, x_{1,} x_{2,}.., x_n)$$

Using the ode Functions

- In order to use ode45 of other ODE solving functions, we must define our function in an m-file because we want to pass a handle to it into the ode function. This function should return the derivatives as a column vector.
- If we don't use any return values the function will simply show a plot. A single return value returns a structure. Having two return values gives us the times as well as the arrays of values at each time. That can then be plotted as we see fit.

Examples

- ODEs abound in physics. The simplest ones involve solving the paths of particles interacting through a force like gravity or having masses on springs.
- Population biology can also be expressed as differential equations. Consider things like predator-prey models where how quickly a population changes depends on how much food it has, how many there are now, and how many predators there are.

Reminders

- Assignment #5 is due next Wednesday.