Symplectic Integrators and N-body

10-6-2006
Do you have any questions about the quiz?
What did we talk about last class?
You have seen that ode45 fails to do a good job of conserving energy in the systems we have given it. We could try to increase the accuracy, but that's just a stop-gap. We really need a different type of integrator.

To understand symplectic integrators we should talk briefly about Hamiltonian systems. They are defined by a value $H(p,q)$ which is basically the total energy of the system in terms of momenta ($p$) and positions ($q$) of the bodies.

For Hook's law $H=0.5*\frac{p^2}{m}+0.5*kq^2$. This is just kinetic plus potential energy. The time derivatives of $p$ and $q$ are given by the partial derivatives of $H$. 
We can build a symplectic integrator by breaking the problem into pieces that we can solve exactly, then alternating between solving those pieces (this is a simplified view). We will use a T+V style also known as a kick-step method.

Given the current position we calculate change in momentum and apply that. Then we take a step using the new momentum. Simply repeat this for the integration.

This will perfectly integrate some Hamiltonian system that is similar to the one we are really interested in.
I want the two of you to pick a Hamiltonian N-body system to integrate and test at least three different integration schemes on it.

- Ode45
- Euler
- Leapfrog

Your write-up should include analysis of how well they conserve the Hamiltonian and how fast they are when you want the Hamiltonian conserved to a certain level.

You can also try seeing what you can do in Matlab to make it more efficient for large N.
No class next week. Write me e-mails with questions.
The test is Wednesday after I get back and the project is due that Friday.