Difference and Differential Equations

2-24-2005

Opening Discussion

Basics of Difference and Differential Equations

- Differential equations describe continuous systems. With these equations, rates of change are defined in terms of other values in the system.
- Difference equations are a discrete parallel to this where we use old values from the system to calculate new values.
- The order of an equation is how many timestep back you are looking at or what the highest power derivative involved is.

Discrete Delays in Code

- Difference equations have an implicit timestep in them. We keep track of variables with integer subscripts and the difference between consecutive subscripts is a timestep.
- You have to be careful when coding solutions to these systems to use the old values in the calculation of all new values.
- Consider the following equations
  - $x_t=0.5*x_{t-1}+0.4*y_{t-1}+0.1$
  - $y_t=0.7*x_{t-1}$

The Logistic Equation

- A very simple example of a difference equation is the logistic equation.
  - $x_t=a*x_{t-1}(1-x_{t-1})$
- This deceptively simple equation holds a significant amount of complexity. Depending on the value of “a” we get different types of behavior.
- The best way to visualize this is actually with a picture plotting how the value of x evolves.
- Fibonacci numbers are another form of difference equation used to model population growth.

Higher Order Difference Equations

- If we have a difference equation with an order higher than 1 then keeping track of previous values can be a difficult task. This can be simplified with the use of a circular queue.
- Basically, keep an array of previous states that holds as many states as the order of the equation (or one more if you do the calculation in the queue). You reference states with an offset and overwrite old values as you go. Remembers mods.

Differential Equations

- Most physical laws are defined in terms of differential equations or partial differential equations. For this reason, being able to solve these is remarkably handy. Unfortunately, they aren't as straightforward as difference equations.
- To solve a differential equation, we basically convert it to a difference equation. That conversion must be done with care though and how we do it depends no how accurate we need our method to be.

Meaning of Derivative and Euler's Method

- Remember from calc 1 the definition of a derivative.

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

- We can use this to directly derive a formula for new values of x based on old values of x and it's derivative.

Higher Order Integrators

- Runga-Kutta stuff here.

Examples

- Predator-prey
- Ballistics
- Harmonic Motion
- Lorenz System

Minute Essay

- How are things going with assignment #3?