Basic Physical Models

3-1-2005

Opening Discussion

- Do you have any questions about the quiz?
- What did we talk about last class?

Solving Differential Equations

- Last time we looked quickly and informally at solving differential equations. Let's take a deeper look today.
- Taylor-Series Method
  - Remember that we can approximate a function near some point t with a Taylor-series.
    - $x(t+h)=x(t)+h*x'(t)+(h^2/2!)*x''(t)+(h^3/3!)*x'''(t)+...$
  - Given a first order differential equation and a starting point we can build an approximate function using this method.
  - The number of terms we use gives the order of our solver.

More Taylor Stuff

- Unfortunately the Taylor-series method requires a lot of analytic derivatives. That's bad for general libraries.
- The Euler method that we talked about last time is a first order Taylor method. It is flexible, but requires that we take really small timesteps.
- Taylor-series methods have errors that we can easily estimate the size of.

Runga-Kutta Methods

- For a Runga-Kutta method we don't take the higher derivatives, instead we approximate them with combinations of f(x,t) values. The derivation of those terms is better suited for the Numerical Analysis course than here.
- The second order Runga-Kutta is generally written as follows with h as the timestep.
  - $x(t+h)=x(t)+1/2(F_1+F_2)$
  - $F_1=h*f(t,x)$
  - $F_2=h*f(t+h,x+F_1)$
- The error is $O(h^3)$.

Fourth-Order Runga-Kutta

- The fourth-order Runga-Kutta integrator seems to be something of a favorite because of the combination of accuracy and simplicity.
  - $x(t+h)=x(t)+1/6(F_1+2F_2+2F_3+F_4)$
  - $F_1=h*f(t,x)$
  - $F_2=h*f(t+0.5*h,x+0.5*F_1)$
  - $F_3=h*f(t+0.5*h,x+0.5*F_2)$
  - $F_4=h*f(t+h,x+F_3)$
- Notice each F is an approximation of a midpoint derivative.

Hamiltonian Dynamics

- Your book doesn't go into this, but a very common method of representing physical systems is Hamiltonian dynamics. In this system you have a function called the Hamiltonian which is the energy for most systems. It is described by variables/vectors called p and q where p is analogous to momentum and q is a position.
  - dq/dt=dH/dp
  - dp/dt=-dH/dq

Multistep Methods

- Some numerical integration techniques are designed so that different parts of a system take different timesteps. This allows the simulation to take large timesteps for things that are changing slowly and small timesteps for things that are changing quickly.
- To implement this type of system you use a priority queue that keeps track of what things need to update and when.

Code to Solve Physical Models

- We are going to do this old-school today. One of you will be doing the typing for writing code that could solve various types of differential equation based systems.
- Preditor-prey
- Ballistics
- Harmonic motion
- Lorenz
- Coupled oscillator

Minute Essay

- What did you think about the style that was done in class today.