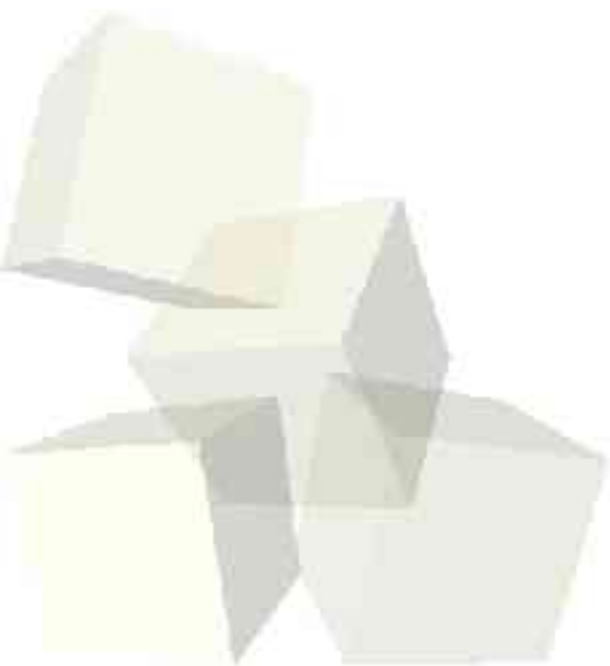




Basic Physical Models

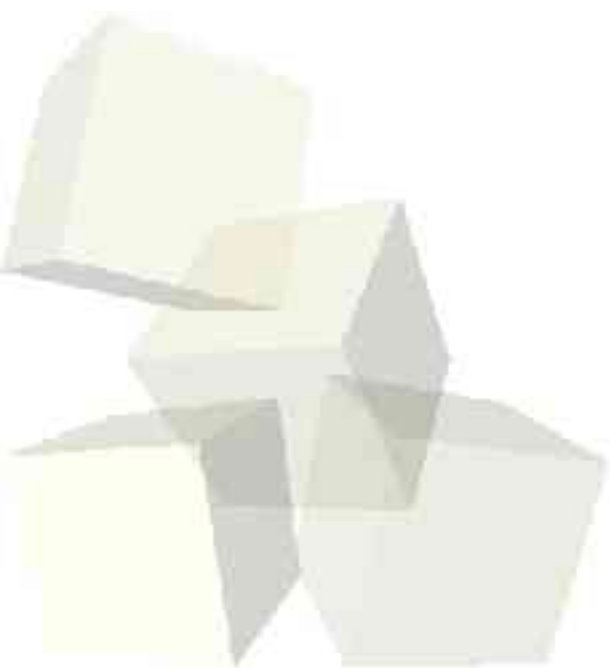
3-1-2005





Opening Discussion

- Do you have any questions about the quiz?
- What did we talk about last class?





Solving Differential Equations

- Last time we looked quickly and informally at solving differential equations. Let's take a deeper look today.
- Taylor-Series Method
 - ◆ Remember that we can approximate a function near some point t with a Taylor-series.
 - $x(t+h) = x(t) + h \cdot x'(t) + (h^2/2!) \cdot x''(t) + (h^3/3!) \cdot x'''(t) + \dots$
 - ◆ Given a first order differential equation and a starting point we can build an approximate function using this method.
 - ◆ The number of terms we use gives the order of our solver.



More Taylor Stuff

- Unfortunately the Taylor-series method requires a lot of analytic derivatives. That's bad for general libraries.
- The Euler method that we talked about last time is a first order Taylor method. It is flexible, but requires that we take really small timesteps.
- Taylor-series methods have errors that we can easily estimate the size of.



Runga-Kutta Methods

- For a Runga-Kutta method we don't take the higher derivatives, instead we approximate them with combinations of $f(x,t)$ values. The derivation of those terms is better suited for the Numerical Analysis course than here.
- The second order Runga-Kutta is generally written as follows with h as the timestep.
 - ◆ $x(t+h)=x(t)+1/2(F_1+F_2)$
 - ◆ $F_1=h*f(t,x)$
 - ◆ $F_2=h*f(t+h,x+F_1)$
- The error is $O(h^3)$.

Fourth-Order Runge-Kutta

- The fourth-order Runge-Kutta integrator seems to be something of a favorite because of the combination of accuracy and simplicity.
 - ◆ $x(t+h) = x(t) + \frac{1}{6}(F_1 + 2F_2 + 2F_3 + F_4)$
 - ◆ $F_1 = h * f(t, x)$
 - ◆ $F_2 = h * f(t + 0.5 * h, x + 0.5 * F_1)$
 - ◆ $F_3 = h * f(t + 0.5 * h, x + 0.5 * F_2)$
 - ◆ $F_4 = h * f(t + h, x + F_3)$
- Notice each F is an approximation of a midpoint derivative.




Hamiltonian Dynamics

- Your book doesn't go into this, but a very common method of representing physical systems is Hamiltonian dynamics. In this system you have a function called the Hamiltonian which is the energy for most systems. It is described by variables/vectors called p and q where p is analogous to momentum and q is a position.
 - ◆ $dq/dt = dH/dp$
 - ◆ $dp/dt = -dH/dq$



Multistep Methods

- Some numerical integration techniques are designed so that different parts of a system take different timesteps. This allows the simulation to take large timesteps for things that are changing slowly and small timesteps for things that are changing quickly.
 - To implement this type of system you use a priority queue that keeps track of what things need to update and when.
- 



Code to Solve Physical Models

- We are going to do this old-school today. One of you will be doing the typing for writing code that could solve various types of differential equation based systems.
- Predator-prey
- Ballistics
- Harmonic motion
- Lorenz
- Coupled oscillator



Minute Essay

- What did you think about the style that was done in class today.

