No Data/Poisson Processes

2/25/2009
Opening Discussion

• What did we talk about last class?
• A fair number of people would like to include more specific examples of simulations and potentially writing of simulations. I can try to do that more in the second half of the semester, but remember to be careful what you ask for.
Dealing with Insufficient Data

- Ask SMEs for what they think the bounds and the mean should be.
- Use triangle distribution to match those numbers.
- Find beta distribution that gives you the right shape.
Poisson Processes

• Arrivals are often well modeled by a Poisson process.

• A Poisson process is defined as having three characteristics.
  – Events happen one at a time.
  – The number of events between \( t \) and \( t+s \) is independent of the number of earlier events.
  – The number of events between \( t \) and \( t+s \) is independent of \( t \).
Properties

• If \( N(t) \) is a Poisson process then the following is true.

\[
P[n(t+s)-N(t)=k] = e^{-\lambda s}(\lambda s)^k \frac{1}{k!}
\]

  For \( k=0,1,2,\ldots \) and \( t,s \geq 0 \)

• If \( N(t) \) is a Poisson process with rate of \( \lambda \) then the interarrival times, \( A_1, A_2, \ldots \) are IID exponentials with mean of \( 1/\lambda \).
Nonstationary Poisson Process

- Throw out the third requirement.
- Let \( \Lambda(T) = \mathbb{E}[N(t)] \).
  \[
  \lambda(t) = \frac{d}{dt} \Lambda(t)
  \]
- Then
  \[
  P[N(t+s) - N(t) = k] = \frac{e^{b(t,s)}[b(t,s)]^k}{k!}
  \]
  \[
  b(t,s) = \Lambda(t,s) - \Lambda(t) = \int_t^{t+s} \lambda(y) dy
  \]
Related Tangent

- This discussion in the book was actually quite enlightening for an article I read in Physics Today recently.
- The article was about light emission from nanoprocesses.
- The distribution of illumination for these is not a exponential. That implies it is not a Poisson process, and that means there is a more complex mechanism involved.
Power-law Distribution

- The distribution of on and off times followed a power-law distribution. This is also a common distribution in many other areas, including planetary science.
- Both differential and cumulative distributions have the same form of $x^{-q}$.
- Particle sizes like to follow this with a differential $q$ of about 3.
Minute Essay

- The midterm is next class. Do you have any other suggestions for questions?
- I'll try to get a review sheet up soon to help you focus your studying, but like my normal tests it can cover anything we have done.
- Feel free to bring a single sheet of paper cheat sheet. I don't know if it will help but it is a good way to study.