

# No Data/Poisson Processes

2/25/2009

# Opening Discussion

- What did we talk about last class?
- A fair number of people would like to include more specific examples of simulations and potentially writing of simulations. I can try to do that more in the second half of the semester, but remember to be careful what you ask for.

# Dealing with Insufficient Data

- Ask SMEs for what they think the bounds and the mean should be.
- Use triangle distribution to match those numbers.
- Find beta distribution that gives you the right shape.

# Poisson Processes

- Arrivals are often well modeled by a Poisson process.
- A Poisson process is defined as having three characteristics.
  - Events happen one at a time.
  - The number of events between  $t$  and  $t+s$  is independent of the number of earlier events.
  - The number of events between  $t$  and  $t+s$  is independent of  $t$ .

# Properties

- If  $N(t)$  is a Poisson process then the following is true.

$$P[n(t+s) - N(t) = k] = \frac{e^{-\lambda s} (\lambda s)^k}{k!}$$

– For  $k=0,1,2,\dots$  and  $t,s \geq 0$

- If  $N(t)$  is a Poisson process with rate of  $\lambda$  then the interarrival times,  $A_1, A_2, \dots$  are IID exponentials with mean of  $1/\lambda$ .

# Nonstationary Poisson Process

- Throw out the third requirement.
- Let  $\Lambda(T) = E[N(t)]$ .

$$\lambda(t) = \frac{d}{dt} \Lambda(t)$$

- Then

$$P[N(t+s) - N(t) = k] = \frac{e^{-b(t,s)} [b(t,s)]^k}{k!}$$

$$b(t,s) = \Lambda(t,s) - \Lambda(t) = \int_t^{t+s} \lambda(y) dy$$

# Related Tangent

- This discussion in the book was actually quite enlightening for an article I read in Physics Today recently.
- The article was about light emission from nanoproceses.
- The distribution of illumination for these is not a exponential. That implies it is not a Poisson process, and that means there is a more complex mechanism involved.

# Power-law Distribution

- The distribution of on and off times followed a power-law distribution. This is also a common distribution in many other areas, including planetary science.
- Both differential and cumulative distributions have the same form of  $x^{-q}$ .
- Particle sizes like to follow this with a differential  $q$  of about 3.

# Minute Essay

- The midterm is next class. Do you have any other suggestions for questions?
- I'll try to get a review sheet up soon to help you focus your studying, but like my normal tests it can cover anything we have done.
- Feel free to bring a single sheet of paper cheat sheet. I don't know if it will help but it is a good way to study.