Opening Discussion

- Minute essay comments:
  - MAS for the project.
  - Proper scope of the projects.
  - Complex projects and safety net.
Convolution

- If $X = Y_1 + Y_2 + ... + Y_n$ and we can generate the $Y$ values we simply do so.
- This is different from composition where the $F$ functions are summed.
This method is somewhat indirect. We generate values and reject them if they aren't good.

Pick majorizing function $t(x) \geq f(x)$ for all $x$. Let $r(x)$ be $t(x)/c$ where $c$ is the integral over $t$.

Process

- Generate $Y$ from $r$.
- Generate $U$ independent of $Y$.
- If $U \leq f(Y)/t(Y)$, return $X = Y$. Otherwise goto 1.
Let's look at a graphical description of what acceptance-rejection is doing.

Note that this works best if $c$ is close to one because we only accept $1/c$ of the generated values.

It is easy to use a constant $t(x)$, but that isn't always efficient.
I'll let you read about this one in the book. I'm not even going to try doing it in class.
Some distributions have other nice properties that we can use to help with generating them.

Often this is a mathematical relationship to some other distribution.

You can view convolution as a type of special property.
Now that we know of several ways to generate continuous random variates we should apply them to the different distributions we looked at back in chapter 6.
Uniform

- $U(a,b) = a + (b-a)U(0,1)$
Exponential

- $X = -\beta \ln U$
m-Erlang

- Generate $U_1, U_2, \ldots, U_m$
- $X = -\beta/m \ln(U_1 \times U_2 \times \ldots \times U_m)$
There are several methods for generating gamma distributions. Because we can't get $F^{-1}(u)$ these are generally acceptance-rejection methods.

Generate $U_1$ and $U_2$.

- $V=a \ln\left[\frac{U_1}{1-U_1}\right]$, $Y=\alpha e^V$, $Z=U_1^2 U_2$, $W=b+qV-Y$.
- If $W+d-\theta Z \geq 0$ return $X=Y$
- If $W \geq \ln Z$ return $X=Y$/ Otherwise start at beginning.
Weibull

- \( X = \beta(-\ln U)^{1/\alpha} \)
Generate $U_1$ and $U_2$. $V_i = 2U_i - 1$, $W = V_1^2 + V_2^2$.

If $W > 2$ return to first step. Otherwise, $Y = \sqrt{(-2 \ln W) / W}$, $X_1 = V_1 Y$, $X_2 = V_2 Y$. 

Lognormal

- $Y \sim N(\mu, \sigma^2)$
- $X = e^Y$
- $Y_1 \sim \text{gamma}(\alpha_1, 1), \ Y_2 = \text{gamma}(\alpha_2, 1)$
- $X = \frac{Y_1}{Y_1 + Y_2}$
Empirical

- How you do this depends on the type of empirical distribution.
- The book presents methods that don't require doing a search through an array.
Questions?

What are your plans for Spring Break?