

# Probability and Statistics

1-31-2011

# Opening Discussion

- What did we talk about last class?

# Random Variables

- This is a sequence of random numbers.
- Discrete
  - The values can only take on a countable number of possibilities.
- Continuous
  - The value comes from a non-countable set.

# Differential Distribution Functions

- Your book calls these probability distribution functions.

- Discrete

- $p(x_i) = P(X = x_i)$

- $\sum_{i=1}^{\infty} p(x_i) = 1$

- Continuous

- $P(X \in [x, x + \Delta x]) = \int_x^{x + \Delta x} f(y) dy$

- $\int_{-\infty}^{\infty} f(x) dx = 1$

# Cumulative Distribution Functions

- Discrete

- $$F(x) = \sum_{x_i \leq x} p(x_i)$$

- Continuous

- $F(x) = P(X \leq x)$  or

- $$F(x) = \int_{-\infty}^x f(y) dy$$

# Mean or Expected Value

- Discrete

- $E(X_i) = \mu_i = \sum_{j=1}^{\infty} p_{X_i}(x_j)$

- Continuous

- $E(X_i) = \mu_i = \int_{-\infty}^{\infty} xf_{X_i}(x) dx$

- Properties

- $E(cX) = cE(X)$

- $E\left(\sum_{i=1}^n c_i X_i\right) = \sum_{i=1}^n c_i E(X_i)$

# Median

- Discrete
  - Smallest  $x$  such that  $F(x) \geq 0.5$
- Continuous
  - $F(x) = 0.5$

# Variance

- $\sigma_i^2 = E[(X_i - \mu_i)^2] = E(X_i^2) - \mu_i^2$
- Standard deviation is square root of variance.
- Properties

$$\text{Var}(X) \geq 0$$

$$\text{Var}(cX) = c^2 \text{Var}(X)$$

$$\text{Var}\left(\sum_{i=1}^n X_i\right) = \sum_{i=1}^n \text{Var}(X_i)$$

# Covariance

- $C_{ij} = E[(X_i - \mu_i)(X_j - \mu_j)] = E(X_i X_j) - \mu_i \mu_j$
- Measures correlation between random variables.
- Independent random variables have  $C_{ij} = 0$ .
- Sign tells you if they are positively or negatively correlated
- Correlation  $\rho_{ij}$  fixes units problem.

# Stochastic Processes

- This is a collection of similar random variables ordered over time. Our simulation outputs can be considered to be stochastic processes.
- Covariance-stationary implies that the mean and variance don't change over time.

# Minute Essay

- Do you have any questions about what we covered today?