Maximum Flow

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Opening Discussion

Johnson's Method

- Last time we mentioned that Johnson's method worked by reweighting all the edges in the graph so none were negative, then used Dijkstra's repeatedly.
- The new weights were given by $w'(u,v)=w(u,v)+h(u)-h(v)$.
- To get the h function the add one new vertex, s, to the graph that has edges out to all other vertices. These edges have weight 0. Then h(v) is set to be the shortest path from s to v. This is found with Bellman-Ford.
- At the end, the shortest path from u to v is what was found on the altered graph $+h(v)-h(u)$.

Flow Networks

- We now switch to graphs that have a slightly different concept behind them. The weighted graphs we had been doing implied some type of distance between vertices. Now we want to have flows between vertices and the weights are called capacities. They are how much can flow through a given connection.
- The flow function has certain constraints.
  - f(u,v)<=c(u,v)
  - f(u,v)=-f(v,u)
  - Sum of flows in and out is 0, except s and t.
- For the maximum flow problem we want to find the largest allowed sum of the flows out of the source.
- Multiple sources and sinks don't add generality.

Ford-Fulkerson Method

- This actually encompasses several different algorithms for doing maximum flow.
- The method is as follows.
  - Initialize the flow to 0
  - While there is an augmenting path, p
    - Augment f with p
  - Return f
- An augmenting path is a path from the source to the sink along which more flow could be moved.
- The residual capacity is simply the capacity minus the flow at any given time. A residual network is a graph that only includes residual edges. That might include some edges not in the original graph because flow goes both ways.

Augmenting Paths

- Once we have a residual network, an augmenting path is simply a path through the augmenting network.
- The smallest edge on the path in the residual network tells us the residual capacity of that path.
- A cut is a partitioning of the graph into sets S and T such that the source is in S and the sink is in T. We can define a net flow and a capacity for a cut. These are the sums of the flows and capacities from any element of S to any element of T.
- A flow is maximal if there are no further augmenting paths on the residual network.

Basic Ford-Fulkerson

- After setting all the flows to 0 do the following.
- While there is a path in the residual network
  - Find the minimum capacity in that path.
  - For each edge in the path increment the flow by that minimum capacity. (And set the opposite direction to negative the flow.)
- If the path is found using breadth first search this algorithm runs in polynomial time. This is the Edmonds-Karp algorithm and it runs in $O(VE^2)$.

Reminders

- Nothing is due soon. I'll get a description of assignment #6 up soon. I have to decide which of the algorithms that we've discussed you should have to write.
- Next week the "test" will be doing the High School programming problems. The question is, when should we plan on starting it?