Matrix Stuff and Linear Prog.

4-4-2006

Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?

Matrices

- You should all know what a matrix is, but let's review, just in case.
- A matrix is a 2-D array values. We talk about them being m x n where m is the number of rows and n is the number of columns.
- Whether you have had linear algebra or not you should read 28.1 as a refresher. As with most things in CLRS they will cover a large fraction of other courses in one section.
- They also have a discussion of a method for doing matrix multiplication in time less than $O(n^3)$. We aren't going to go through that in class.

Systems of Linear Equations

- Many problems can be expressed in terms of systems of linear equations. We have a number of equations of the form $a_{11}x_1+a_{12}x_2+...+a_{1n}x_n=y_1$. How many equations we have determines whether our problem is overdetermined, underdetermined, or properly determined. We can express this in the form $Ax=b$ and we want to solve for x.
- The obvious solution is to find A^{-1} and say $x=A^{-1}b$. However, it turns out this method isn't that numerically stable and has other drawbacks.

LUP Factorization

- The solution we take instead is to factorize A into a form that can be solved easily. There are many ways of doing this. We will discuss LUP factorization. The idea is to find matrices L, U, and P such that PA=LU and such that we can easily solve the equation LUx=Pb.
- We make L and lower triangular unit matrix, U and upper triangular matrix, and P a permutation matrix. Solving with triangular matrices is much easier than with full matrices.
- So we first solve Ly=Pb, then solve Ux=y. The trick is getting L, U, and P.

Finding L, U, and P

- The way we do this is basically to use Gaussian elimination. This process goes through and eliminates elements from a matrix by adding multiples of some rows to others.
- That gives L and U. P is required for numerical stability. The LU factorization requires dividing by elements and we want to divide by the largest element possible. To get this we permute the matrix so that the column with the largest element on the diagonal is the one we divide by.
- CLRS gives a wonderful algorithm for this on page 752 that calculates L and U in place in A and simply doesn't bother with the elements we know the value of.

Linear Programming

- In general, this is the process of trying to optimize a linear function while staying within a set of linear constraints. The constrains can be either linear equalities or linear inequalities (<= or >=).
- This can be done in polynomial time, but is often done with the simplex algorithm. It isn't polynomial in the worst case, but it is well behaved in most cases and it has low overhead.
- Standard form for linear programming involves maximizing a function while maintaining a set of linear inequalities. Slack form would instead use linear equalities.

More Linear Programming

- And value of the variables that satisfies the constraints is called a feasible solution. The set of all feasible solutions produces a feasible region. For standard form this will be a convex space.
- In theory we could evaluate the objective function (the function we want to maximize) at every point in the feasible region. It turns out that optimal solutions can only be on boundaries of the constraints. In fact, they will only be at intersections of boundaries of constraints.
- The feasible region is the intersection of many half-spaces and it is called the simplex. The simplex algorithm starts at one vertex and moves along edges of the simplex.

Reminders

- Because 2 members of the class are out of town on Thursday I'm going to push back the due date for assignment #6 until next Tuesday. That way everyone has equal footing.