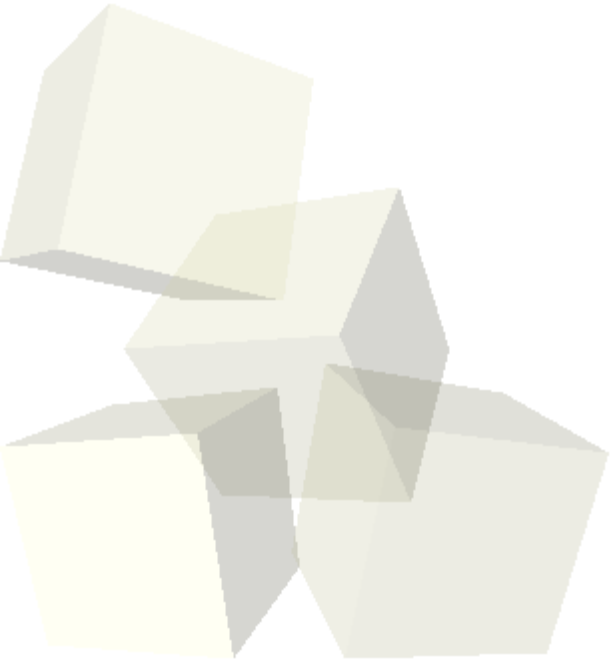
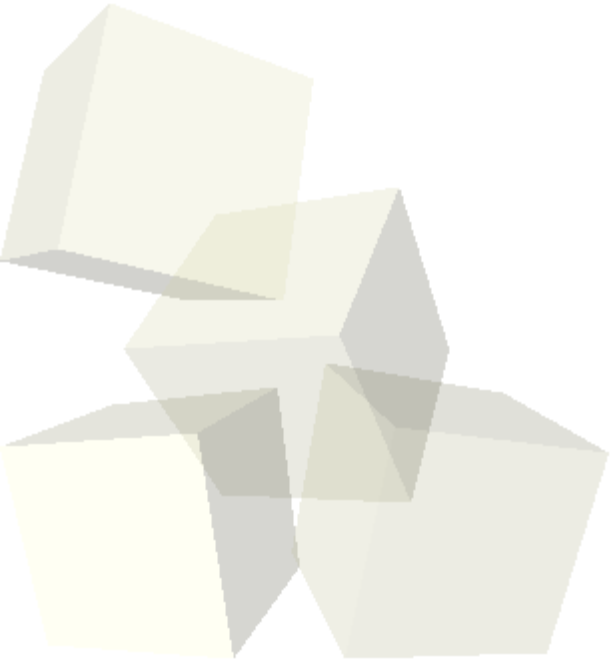
Linear Prog. And String Matching

4-6-2006



Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?



Linear Programming

- Recall that the definition of a linear program is a linear equation that we want to optimize along with a set of constraints provided by linear equalities or inequalities.
- The region of feasible solutions that satisfy the constraints is called the simplex and it is always a convex hull in the space of the problem.
- The way the simplex algorithm works is to move from one vertex on the hull to another until it finds the largest one.

Doing Simplex

- The functioning of the simplex algorithm is to convert to a slack form with linear equalities where some variables, called basic variables, are set in terms of other variables, “nonbasic variables.”
- We move from one vertex to another by changing one basic variable to a nonbasic variable and changing one back the other way. This operation is called a pivot.
- Ellipsoid and interior-point algorithms happen in polynomial time. When only integer values are allowed we have integer linear programming which is NP hard.

Algebraic Statement

- You are given a m by n matrix A, a m-vector b and and n-vector c. You want to find the n-vector x that maximizes $c^T x$ and satisfies Ax<=b for all elements in Ax and b. All elements of x must also be non-negative.
- Converting to slack form we make it so only the non-negativity constraints are inequalities and all others are equalities. To do this we introduce new variables of the form $x_{n+i}=b_i-\text{sum}(a_{ij}x_j)$. The lower x values are the basic variables and the higher ones are the nonbasic variables.

Problems as Linear Programs

- Consider the problem of single-pair shortest-path on a graph.
  - Objective function is d[t].
  - Constraints are d[v]<=d[u]+w(u,v) for all edges.
  - d[s]=0.
- Maximum flow can also be solved with linear programming
  - Objective function is sum(f(s,v)).
  - Constraints
    - f(u,v)<=c(u,v)
    - f(u,v)=-f(v,u)
    - sum(f(v,u))=0
  - The form of this can be optimized.

Reminders

- The assignment is due next Tuesday.