Computational Geometry

4-13-2006

Opening Discussion

- What did we talk about last class?
- Do you have any questions about the assignment?
- Do you have any questions about the test?

Computational Geometry

- This is the field of algorithms associated with geometric objects distributed in space.
- CLRS only deals with 2-D objects because they are a lot easier to illustrate. However, the area of computational geometry includes higher dimensions as well.
- In class we will go over things in both 2 and 3 dimension just because it is fun. Today we will be laying out some of the mathematical foundations for doing computational geometry and look at some simple algorithms.
- Good algorithms will try to avoid using trig functions and even division as that improves speed and numerical stability.

Primitives

- Points – A location specified by n scalars for n dimensional space.
- Lines – Given two points, p_1 and p_2, this is all points $p=ap_1+(1-a)p_2$.
- Segments – The piece of a line between two points. Limit the a value to between 0 and 1.
- Rays/Directed Segments – Sometimes we care about the ordering of the points on a segment or we care about a values greater than 0.
- Vectors – A directed segment with the first point at the origin is a vector. It is not uncommon to swap between points and vectors. The point is the location and the vector is the direction to it.

Cross Product

- A cross product is an operation on two vectors.
- In 3-D the cross product gives a vector that is perpendicular to the two other vectors and has a magnitude equal to the sin of the angle between them times their magnitudes.
- CLRS presents a cross product in 2-D that is the signed area of a trapozoid. It turns out that it is the same value, but the perpendicular vector interpretation doesn't make since in 2-D.
- The cross product can give the relative orientation of points in space. I find the 3-D analogy and the right-hand rule easiest for remembering this.

Dot Product

- CLRS doesn't introduce the dot product, but it is very helpful when working in 3-D. It can be helpful in 2-D, but not as much so.
- The dot product, or inner product, is given by the sum of the products of the terms of two vectors and always returns a scalar.
- That scalar value is equal to the cos of the angle between the vectors times the magnitudes of the vectors.
- In 3-D the dot product is good for defining distances from a plane (or planes themselves). In 2-D you get distances from lines or new parallel lines.

Intersecting Segments

- You can use the cross product to fairly easily determine if two segments intersect. If each segment spans the line of the other then they intersect.
- To find this you cross the direction vector for one segment with the direction from that segment to the ends of the other segment. If those cross products both have different signs then you have a crossing.
- You also need to check for the boundary condition where a point lies on the other segment.

Reminders

- Have a good weekend.