Solar System Astronomy Midterm

1. Describe why we have spring and neap tides. What lunar phases do these happen at? Draw a picture to illustrate your point.

The Earth feels tidal forces from both the Moon and the Sun. When those line up we have a spring tide. When they are perpendicular we have a neap tide. You should draw something like figure 4.25 on page 141 in the book.

2. How many protons, neutrons, and electrons are in each of the following?
a. Helium with atomic number=2, atomic mass=4, neutral charge?

2 protons, 2 neutrons, 2 electrons

b. Carbon with atomic number=6, atomic mass=12, singly ionized?

6 protons, 6 neutrons, 5 electrons

This material was covered on page 160-161 in the text.

3. You have a friend who lives in Thomas and every morning he eats a doughnut and drinks a Mountain Dew. Having taken much criticism for the number of empty calories in this breakfast (400 Cals), he decides that after breakfast he will jog up the stairs from the ground floor to the 8^{th} floor to burn off the calories. After each trip he takes the elevator down since that direction is a lot harder on the knees. You are going to help him figure out how many flights he has to climb to burn off that breakfast. The stairs in Thomas are a single flight from one floor to the next. We will estimate that each flight is 4 meters in elevation. Not only will you take the change in height into account, but also the changes in speed, though we will ignore any internal inefficiencies and only consider straight potential and kinetic energy. Assume your friend reaches a maximum speed of 5 m/s on each flight and comes to a complete stop at the landings. Your friend has a mass of 100 kg. (Hint: figure out how much energy is burned on each flight, both potential and kinetic, then divide that into the energy in the breakfast.)

Given this description the first thing to do is figure out how much energy he puts into kinetic and potential energy for each flight. The assumption that kinetic and potential energy aren't linked isn't perfectly realistic, but it helps compensate for the fact that we are ignoring friction and other factors.

$$E_{potential}=mgh=100[kg]*9.8[m/s^2]*4[m]=3920[J]$$

$$E_{kinetic}=0.5*m*v^2=0.5*100[kg]*(5[m/s])^2=50*25[J]=1250[J]$$

We need to just convert the sum into Cals and divide that into 400.

$$(3920[J]+1250[J])*\frac{1[Cal]}{4184[J]}=1.24[Cal]$$

$$400[Cal]/1.24[Cal/flight]=323\ flights$$

He's going to be doing a lot of climbing. Your answer might be a bit different if you rounded values.

For some reason, many people in the class went with the formula $E=mc^2$. That is the formula behind atomic bombs, not stair climbing. It is unclear why so many people wanted to blow up their friend.

4. Why is it so hard to walk on ice? Cast your answers in terms of the physics of linear motion.

It is difficult to walk in ice because the low coefficient of friction makes it very difficult for you to transfer momentum to the Earth. Newton's laws tell you that an object won't change it's motion unless acted on by an outside force and that for any force there has to be an equal an opposite force. Those both relate to the fact that momentum is conserved. You can cast your answer in terms of any of those. The real problem is that normally when you walk you push on the Earth and it pushes back. In this process you get momentum in one direction by giving the Earth an equal amount of momentum in the opposite direction. When you are standing on ice there isn't much friction between your feet and the Earth so you have a very hard time pushing laterally on the Earth and the Earth doesn't push back laterally on you much either.

5. For this problem and the one after it we will consider the star HD72659, which has a planet in orbit around it. The planet orbits the star at a distance of 4.5 AU. The system is 168 ly from the Earth. How large of a telescope would you need to be able to resolve the planet as being separate from the star looking in 500 nm light (assume the atmosphere isn't a problem so you only deal with the diffraction limit).

This is probably the hardest question on the exam. To answer this equation you have to use two different formulas and really understand what they mean. This said, it is simply an application of Math Insights 6.1 and 6.2 in the text. One formula is the formula for angular separation and the other is the formula for the diffraction limit. Both give you angles so you simply put them in the right units and set them equal then solve for the one value that I didn't give you. You have to be careful with the units as you do this as well. First let's get the angular separation of the planet and star.

$$\alpha=\frac{s}{2\pi d}*360^\circ$$

$$\alpha=\frac{s}{2\pi d}*360^\circ*\frac{3600[arcseconds]}{1^\circ}$$

$$\alpha=\frac{4.5[AU]}{2\pi 168[ly]}*360^\circ*\frac{3600[arcseconds]}{1^\circ}$$

$$4.5[AU]*\frac{1.5*10^8[km]}{1[AU]}=6.75*10^8[km]$$

$$168[ly]*\frac{9.46*10^{12}[km]}{1[ly]}=1.59*10^{15}[km]$$

$$\alpha=\frac{6.75*10^8[km]}{2\pi 1.59*10^{15}[km]}*360^\circ*\frac{3600[arcseconds]}{1^\circ}=0.0876[arcseconds]$$

Notice that I multiply by 3600 to get arcseconds since that is what the diffraction limit formula wants. Also notice that I had to get some of the distance units to be the same so they would cancel. Now we can set this equal to the diffraction limit and solve for that aperture size.

$$\alpha=2.5*10^5*\frac{\lambda}{L}[arcseconds]$$

$$0.0876[arcseconds]=2.5*10^5*\frac{\lambda}{L}[arcseconds]$$

$$L=\frac{2.5*10^5*\lambda}{0.0876}$$

$$L=\frac{2.5*10^5*5*10^{-7}[m]}{0.0876}$$

$$L=1.42[m]$$

That is our final answer. Notice that isn't a large telescope by modern standards.

6. Large ground telescopes should be able to resolve the planet as separate from the star. They can't take pictures of the planet though. Why do you think this is? (Hint: What is different about the light from the star and the light from the planet?)

So planets around this star and others are theoretically discernible with existing telescopes like the Keck telescopes. That doesn't work because of both of the things we learned about thermal radiation. The planet only reflects light in the visible and is MUCH dimmer than the star it orbits. It is better if you look in the IR, because you know that the peak emission of the planet is in that region of the spectrum. However, since total emission goes as σT^4, you know that the star is going to outshine the planet by a significant amount not matter what frequency you look at it in.

7. In your future life you become a writer for a reality show called “Fear Factor: Space”. In one of the episodes you are going to make people jump off high structure straight down to rock, without a safety net (they will have space suits on, but those won't protect against broken bones). You have to figure out how high you can have people jump from on different bodies without too much of a risk of them being seriously injured. (This information is used for the next problem as well.)

First you need to figure how fast they can be moving when they hit the ground and not injure themselves. The suits are fairly massive so you figure jumping from about 2 meters would be the highest they could safely jump from on the Earth. How fast will they be moving when they hit the ground? (Hint: Find the potential energy of being 2 meters up at the surface of the Earth, then figure out at what speed you have that much kinetic energy. Also, while you don't have to use this, $R_{Earth}=6.4*10^6$ [m], $M_{earth}=6*10^{24}$ [kg])

The easy way to do this, as the hint implies, is to just set the kinetic energy equal to the potential energy and solve for v. Because we know g for the Earth and we are talking about a very small height compared to the radius of the Earth we can use E=mgh.

$$\frac{1}{2}mv^2=mgh$$

$$v^2=2\text{gh}=2*9.8[m/s^2]*2[m]$$

$$v=\sqrt{39.2[m^2/s^2]}=6.26[m/s]$$

Most people could safely jump to the ground from 3 or 4 meters, but if the suit weighs very much then hitting the ground at around 13 mph is not a bad estimate to keep you safe so you keep your job.

8. How high could they jump from if the set was moved to the surface of the Moon? ($R_{Moon}=0.27R_{Earth}$, $M_{moon}=0.012M_{Earth}$) (Hint: Use Newton's gravity formula to get the acceleration due to gravity at the surface.)

This is harder because we don't know g for the Moon. So our first step is to estimate g on the moon with the help of the radius and mass given above. Remember that g is the acceleration due to gravity. We know that F=ma and for gravity we know the formula for F. So g is Newtonian gravity with one of the masses divided out. The hard way to do this is to plug in the full numbers into Newton's law of gravity like this.

$$g=\frac{GM}{d^2}=\frac{6.67*10^{-11}*0.012*6*10^{24}[kg]}{(0.27*6.4*10^6[m])^2}=\frac{4.8*10^{12}}{3*10^{12}}=1.6[m/s^2]$$

This is an acceptable approach, but it isn't ideal because we have a lot of messy numbers. A shorter approach is to notice that g changes as M/d^2 and start with g for Earth and multiple by the relative changes. That looks like this.

$$g=9.8[m/s^2]*\frac{0.012}{0.27^2}=9.8[m/s^2]*0.165=1.6[m/s^2]$$

To make this more clear, I'll write everything out. The thing to note is that I don't have to use G at any point if I just play with the algebra and treat this as a scaling problem.

$$g_{Moon}=\frac{GM_{Moon}}{r_{Moon}^2}=\frac{G\times 0.012\text{M}_{Earth}}{(0.27\text{R}_{Earth})^2}=\frac{0.012}{0.27^2}\times\frac{GM_{Earth}}{R_{Earth}^2}=0.165\times g_{Earth}=0.165*9.8[m/s^2]=1.6[m/s^2]$$

Now we can repeat the process from the previous question, but here we know the value for v and want to solve for the height.

$$\frac{1}{2}mv^2=mgh$$

$$h=\frac{v^2}{2\text{g}}=\frac{(6.26[m/s])^2}{2*1.6[m/s^2]}=12.24[m]$$

That's a much more impressive height and most people would normally be quite afraid to jump from that high onto stone. Were they on the Moon, however, it should be just as safe as jumping off a 2[m] platform on the Earth.

In case anyone wonders if we really covered enough for you to calculate g for a body, check out the Mathematical Insight on page 143.

9. As we will discuss shortly after the midterm, stars form when giant clouds of gas collapse under their own gravity. Inevitably, these clouds are spinning ever so slightly. Assume a cloud, 1 [ly] across collapsed to form a star. The cloud was originally spinning quite slowly at 10^{-6} [radians/year], so it would take over 6 million years for it to spin around once completely. Were that cloud to collapse to the size of our Sun ($7*10^5$ [km] in radius), how fast would material on the surface at the equator be moving? (In case you don't remember this, a full circle has 2π radians in it and an arc that spans 1 radian has a length equal

to the radius of the circle. This implies that the stuff at the outer edge of the cloud is originally moving at 10^{-6} [ly/year].)

This problem is just a conservation of angular momentum problem. You know that L=mvr and that it has to be conserved as the cloud shrinks (that means that r gets smaller). Notice that you are never given a mass. That's fine though because it cancels out. You are given the starting r and v and the final r. The question is, what is the final v.

$$m v_i r_i = m v_f r_f$$

$$10^{-6}[ly/yr]*1[ly] = v_f * 7*10^5[km]$$

$$v_f = \frac{10^{-6}[ly/yr]*1[ly]}{7*10^5[km]} * \frac{9.46*10^{12}[km]}{1[ly]}$$

$$v_f = 1.35*10^7*10^{-6}[ly/yr]*\frac{9.46*10^{12}[km]}{1[ly]}*\frac{1[yr]}{8760[hr]}$$

$$v_f = 1.46*10^{10}[km/hr] = 4*10^6[km/s]$$

The astute observer will notice that this is higher than the speed of light and hence conclude that something is obviously wrong. That's part of the point though. Even though the original cloud was spinning extremely slowly, it can't all collapse to the size of a star and conserve angular momentum. That is why protostars have disks that planets can form in.

10. Some of the allowed energy levels of singly ionized helium are 6.04 [eV], 3.4 [eV], and 2.176 [eV]. There are three possible energy transitions between these states. What wavelengths of photons would you expect to get from these transitions? Are any of them visible? (Since you probably didn't write this one down, I'll tell you 1 [eV]=$1.6*10^{-19}$ [J]. Also, visible light have wavelengths between 380 and 750nm.)

You have energies and are asked for wavelength. So you want to use E=hc/λ. The first thing to notice is that I give you electron energy levels so the photons are emitted on transitions between these levels. After that just convert eV to J and plug the numbers through.

$$\lambda = \frac{hc}{6.04[eV]-2.176[eV]} = \frac{6.626*10^{-34}[J*s]*3*10^8[m/s]}{6.04[eV]-2.176[eV]}*\frac{1[eV]}{1.6*10^{-19}[J]} = \frac{1.24*10^{-6}[m*eV]}{3.864[eV]} = 3.2*10^{-7}[m] = 320[nm]$$

$$\lambda = \frac{hc}{6.04[eV]-3.4[eV]} = \frac{6.626*10^{-34}[J*s]*3*10^8[m/s]}{6.04[eV]-3.4[eV]}*\frac{1[eV]}{1.6*10^{-19}[J]} = \frac{1.24*10^{-6}[m*eV]}{2.64[eV]} = 4.7*10^{-7}[m] = 470[nm]$$

$$\lambda = \frac{hc}{3.4[eV]-2.176[eV]} = \frac{6.626*10^{-34}[J*s]*3*10^8[m/s]}{3.4[eV]-2.176[eV]}*\frac{1[eV]}{1.6*10^{-19}[J]} = \frac{1.24*10^{-6}[m*eV]}{1.224[eV]} = 10^{-6}[m] = 1000[nm]$$

The middle of these three is in the visible and should be something of a cyan type of color as it is between the wavelengths of blue and green.

Extra Credit: The concept of a space elevator is something that has gained significant support in the last decade as the ability to make structures from carbon nanotubes as improved. This ability is critical since the biggest problem is finding a material strong enough to support its own weight. Steel falls far short of this for a chord as long as is needed. For this problem, you will calculate why people like the idea of a space elevator from an economics point of view. It would be expensive to build one, but once in place, getting into space would be a lot cheaper.

a. We will assume that the primary cost of getting into space on a space elevator is the energy needed to climb the rope. This isn't a bad assumption. The space elevator goes from the surface of the Earth up to geosynchronous orbit (a station is placed there though the cable goes out twice that far). To answer this question first figure out the height above the surface of the Earth for geosynchronous orbit (that is the height at which the orbital period is 24 hours). Recall from one class day that $v_{circular} = \sqrt{GM/d}$.

There are several ways to solve this. Using the formula above you could find the distance, d, at which the velocity is such that you go around in 24 hours. Alternately, you could use Newton's version of Kepler's 3rd law to find the distance at which the period was 1 day. Let's do the first method first.

$$v_{circular} = \sqrt{GM/d} = \frac{2\pi d}{24[hours]}$$

$$GM/d = \left(\frac{2\pi d}{24[hours]}\right)^2$$

$$d^3 = \frac{GM*(24[h])^2}{4\pi^2} = \frac{6.67*10^{-11}[m^3/(kg*s^2)]*6*10^{24}[kg]*(24[h])^2}{4\pi^2}*\left(\frac{3600[s]}{1[h]}\right)^2 = 7.53*10^{22}[m^3]$$

$$d = 4.2*10^7[m] = 42000[km]$$

If instead you use Newton's version of Kepler's 3rd law you have to notice that the object in orbit is much smaller than the Earth so M_1+M_2 can be approximated very well as the mass of the Earth.

$$p^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3$$

$$a^3 = (24[h])^2 * \frac{GM}{4\pi^2}$$

Notice that's the same thing we had above in the 3rd line since a=d.

b. Once you have that, calculate the difference in the gravitational potential energy from the surface of the Earth to that orbit. (One of the earlier questions provides the radius of the Earth if you don't have it written down.)

It would be nice if we could use E_P=mgh here, but we can't. We have gone too far about the surface of the Earth. As such, we have to use the Newton form of potential energy and take the difference for two different heights.

$$E = E_{surface} - E_{geosynchronous} = -\frac{GM_1 M_2}{6.4*10^6[m]} + \frac{GM_1 M_2}{4.2*10^7[m]}$$

$$E = 6.67*10^{-11}[m^3/(kg*s^2)]*6*10^{24}[kg]*M_2*\left(-\frac{1}{6.4*10^6[m]} + \frac{1}{4.2*10^7[m]}\right)$$

$$E = 4*10^{14}[m^3/s^2]*M_2*1.3*10^{-7}[m^{-1}] = 5.2*10^7[m^2/s^2]*M_2$$

We'll have to assume a mass, but it will be 52 million Joules per kilogram that we send up.

c. Now to get a cost, we need to convert that energy to units of kilowatt-hours (1 [kilowatt-hour]=3.6*10^6 [J]). We'll assume you can buy energy at $0.10/kilowatt-hour.

So ow much does that cost? Well, 52 million Joules is 14.4 kilowatt-hours. At the quoted rate that would be a mere $1.44/kg. So putting a human up costs $144 if you don't send any supplies or the car to hold him/her. Considering that current space launches involving humans cost hundreds of millions of dollars, the thought of being able to do it for a few thousand dollars seems quite remarkable.

Extra Credit 2: Doing this problem, you will come up with a formula for the Roche limit. This is the distance away from a planet that two objects must be to stick together gravitationally. Inside of this limit, the tidal force is greater than the attractive force between two small spheres of a given density. Deriving this formula is fairly easy. Assume you have two small spherical masses both of radius r and density ρ. The closer one is at a distance d from the center of the planet while the other is at d+2r. Set the force of attraction between the two equal to the difference in the force of attraction that the two bodies feel from the large central body which has a mass M. Simply solve for d in that equation. You will be able to simplify at some times by using the fact that r is much smaller than d.

This problem is all formula manipulation. We are going to set two forces equal. One is the force of gravity between two small spheres that are touching one another. The other is is difference in the force of attraction those two spheres feel from a central body if one is positioned radially outside the other.

$$\frac{GM_s M_s}{(2r)^2} = \frac{GM_p M_s}{d^2} - \frac{GM_p M_s}{(d+2r)^2}$$

I'm using M_s as the mass of the two spheres and M_p as the mass of the planet. r is the radius of the

small spheres and d is the distance of the inner sphere from the planet. We don't want this to be a function of the mass of the sphere, but it does need to be a function of the density, ρ. So we will substitute density times volume in place of the mass of the spheres. First we will cancel out everything that all the terms share then arrange so we are solving for d.

$$\frac{M_s}{(2r)^2}=\frac{M_p}{d^2}-\frac{M_p}{(d+2r)^2}$$

$$\frac{M_s}{(2r)^2}=\frac{M_p(d+2r)^2-M_p d^2}{d^2(d+2r)^2}$$

$$\frac{(4/3)\pi r^3\rho}{(2r)^2}=\frac{M_p(d^2+4dr+4r^2)-M_p d^2}{d^2(d+2r)^2}$$

$$\frac{(4/3)\pi r^3\rho}{(2r)^2}=\frac{M_p(4dr+4r^2)}{d^2(d+2r)^2}$$

$$(1/3)\pi r\rho=\frac{4M_p r(d+r)}{d^2(d+2r)^2}$$

$$\pi\rho=\frac{12M_p(d+r)}{d^2(d+2r)^2}$$

At this point we will employ our approximation that d is much bigger than r so that d+r and d+2r are both assumed to be d. Obviously we couldn't do this at the beginning else the right side would have been 0. At this point it is a safe approximation though and it allows us to get a final answer.

$$\pi\rho=\frac{12M_p d}{d^4}$$

$$\pi\rho=\frac{12M_p}{d^3}$$

$$d^3=\frac{12M_p}{\pi\rho}$$

This is slightly different from what you would find in Wikipedia or some other sources because they set up the formula a bit differently and use the density of the central body as well. They typically solve for d as a multiple of the planet radius to do that. This solution works for our purposes and can help you to understand why some planets have rings and why the rings haven't clumped together to form moons.