**Game-theoretic Randomization for Security Patrolling with Dynamic Execution Uncertainty**

Albert Xin Jiang*, Zhengyu Yin†, Chao Zhang*, Sarit Kraus*, Milind Tambe*

*University of Southern California, †Bar-Ilan University

---

**Motivation**

**Time-critical** security patrolling domains

Game-theoretic Model for Security: Stackelberg Equilibrium
- Defender commits to a randomized patrol schedule
- Attacker plays best response

Fare Evasion Problem in LA Metro
- In 2007 alone, estimated revenue loss of $5.6 million
- Los Angeles Sheriff’s Department (LASD) periodically patrols the Metro system
- TRUSTS system for randomized fare inspection (2012)

---

**Problem Statement**

Field Tests for TRUSTS v.1 (2012): officer often deviate from schedule (missing a train, making an arrest, etc.)

Execution uncertainty at earlier time steps can affect the defender units’ ability to carry out their planned schedules in later time steps

Desired patrol schedules should be robust against execution uncertainty
- Contain contingency plans

---

**Model**

Patrolling game with execution uncertainty
- Two-player Bayesian Stackelberg game
- Leader (defender) has multiple units
- Defender’s strategy space: an MDP for each unit
- Defender commits to mixed patrol schedule, attacker responds (Strong Stackelberg Equilibrium)
- Multiple types of attacker

States: (location, time)
- Action a
- Transition function T(s,a,s’)
- Utility depends on:joint trajectory of defender units attacker type and action

---

**Computation**

Challenge: exponential # of defender pure strategies

If utility function has separable structure
- Utility decomposed into sum over individual transitions
- Expected utility only depend on the marginal coverage x(s,a,s’)
- Compactly represent defender strategies using marginal coverage
- Standard SSE formulation: efficient practical algorithms (e.g., Yin & Tambe 2012)

For zero-sum games: linear program

\[ \max_{u,\pi} \sum_{s \in S} \sum_{a \in A} \sum_{s' \in S} u(s) x(s,a,s') \pi(s',a) - \sum_{s \in S} \sum_{a \in A} \sum_{s' \in S} x(s,a,s') w(s) \]

- Calculate decoupled Markovian randomized strategy from the marginals

\[ \pi_t(s_t,a_t) = \frac{\exp(x(s_t,a_t))}{\sum_{s',a'} \exp(x(s',a'))} \]

- Sample a deterministic strategy by sampling an action at each state
  - Results in a deterministic MDP policy for each unit
  - Prescribes action at every state, i.e., contingency plan for all situations

---

**Apply to LA Metro**

- Zero-sum
- Approximate utility as separable function

Fare evaders
- Patrol units

---

**Evaluation**

Markov strategy (TRUSTSv2) outperforms TRUSTSv1 with simple contingency plans

---

**Future Work**

- Learning transition probabilities from data
- Non-separable utility: applying techniques from (decentralized) planning under uncertainty / multi-agent coordination, e.g., Dec-MDPs

---

**Mobile Phone Application**

- Stores and visualizes sampled schedule with contingency plan
- Collects data
- Under evaluation by LASD
- Check out our demo on Thursday, 10-11am, 3:30-4:30pm (Luber, Yin, Delle Fave, Jiang, Tambe & Sullivan)

---

**Contributions**

General Stackelberg game model for patrolling with execution uncertainty
- Using Markov Decision Processes to model probabilistic transitions in defender’s execution of patrols
- Combines game theory and planning under uncertainty

Efficient algorithm when utility functions are separable

Outputs robust patrol schedules with contingency plans
- Applied to TRUSTS system for LA Metro
- Smart-phone app under evaluation (See Our Demo!)

---

**Acknowledgements**

We thank the Los Angeles Sheriff’s Department for their exceptional collaboration.