Game-theoretic Randomization for Security Patrolling with Dynamic Execution Uncertainty

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TEAMCORE
USC

Time-critical Security Patrolling Games

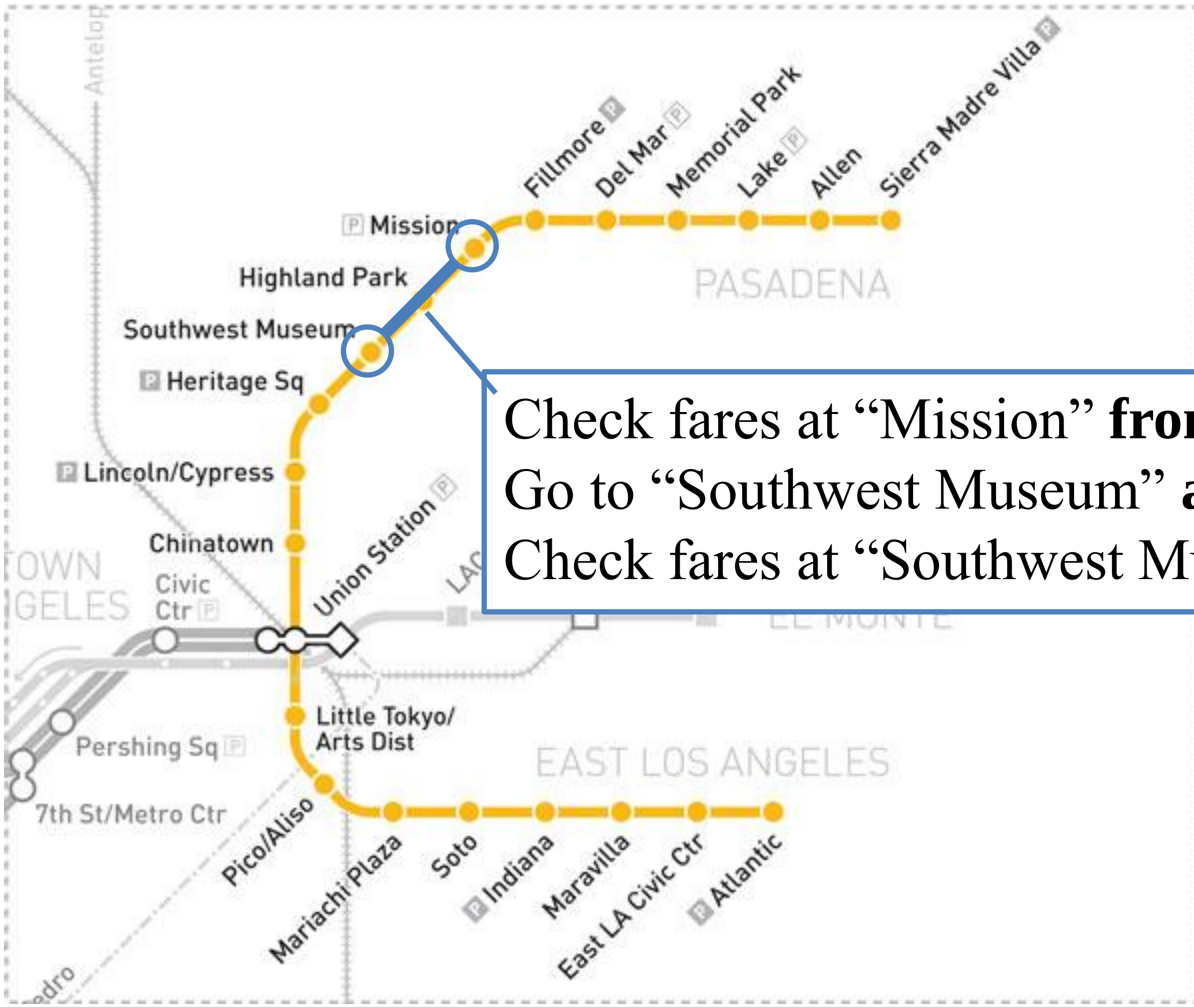
- **Timing** affecting patrol effectiveness

TRUSTS:
Fare inspection in
LA Metro Rail
(Yin et al, 2012)

Ferry escort in New York
(Fang et al, 2013)

PROTECT:
Patrolling Port of Boston
(Shieh et al, 2012, 2013)

TRUSTS: Randomized Fare Inspection



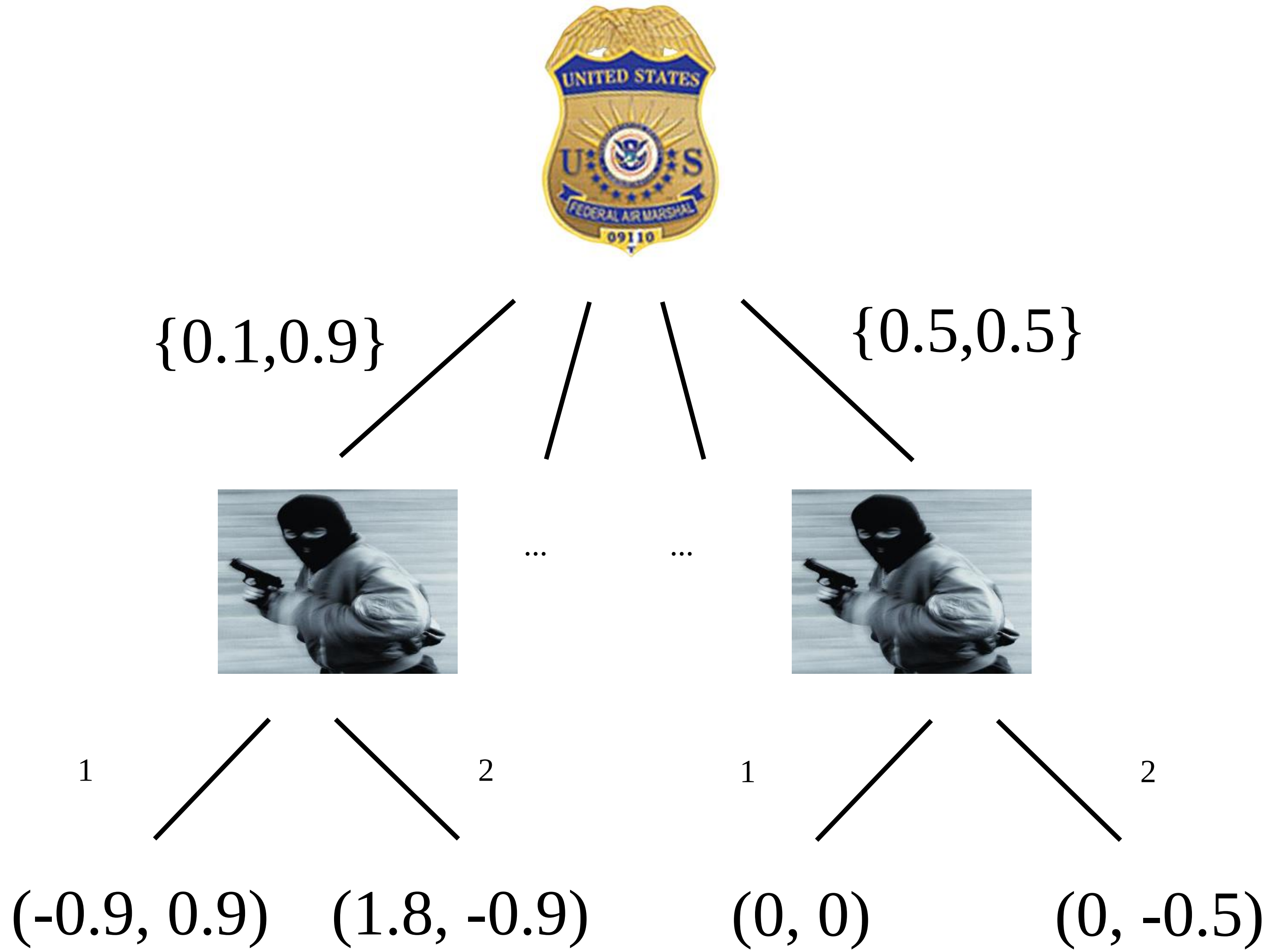
Check fares at "Mission" **from 7am to 7:50am**
Go to "Southwest Museum" **at 7:50am**
Check fares at "Southwest Museum" **from 8am to 9am**

- In collaboration with LA Sheriff's Dept (Yin et al 2012)

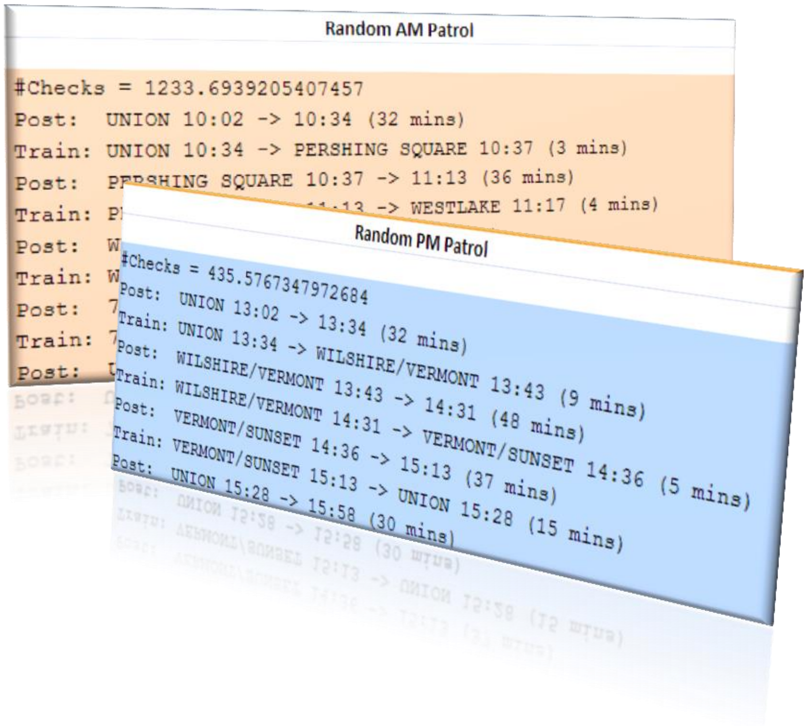
Stackelberg Equilibrium

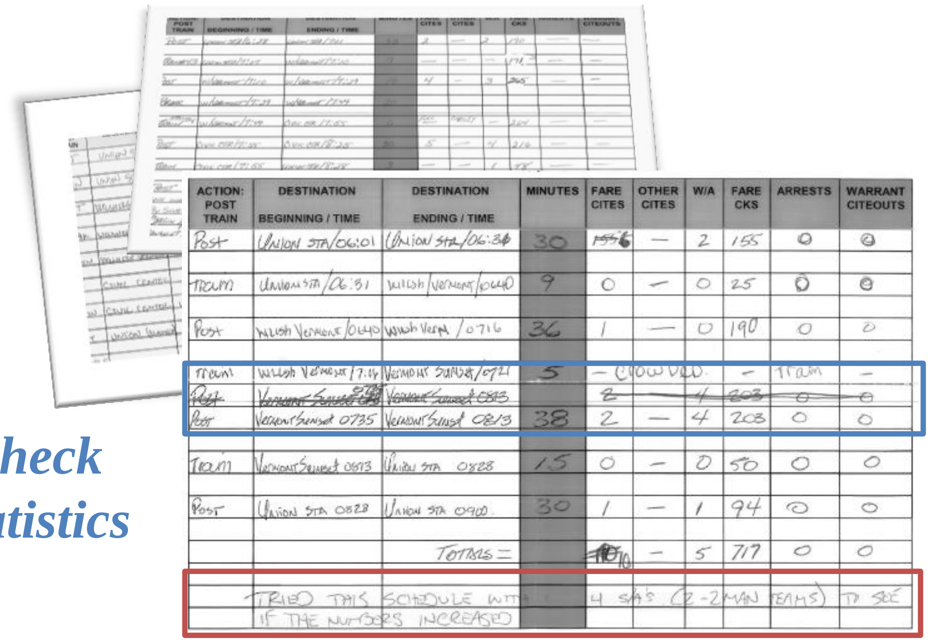
Attackers use surveillance in planning attacks

Defender commits to a mixed strategy



Field tests of TRUSTSv1 (2012)





Check Statistics

Remarks

Feedback: officer often deviate from schedules

- *Felony arrest*
- *Called to deal with emergencies*

Problem: Dynamic Execution Uncertainty

- Execution uncertainty can affect the defender units' ability to carry out their planned schedules in later time steps
- Want robust patrol schedules with contingency plans
- Related work on uncertainty in security games
  - *either only applicable to static domains* (Yin et al 2011; 2012)
  - *or consider general dynamic game formulations (NP-hard)*
    - (Letchford et al 2010; 2012; Vorobeychik et al 2012)

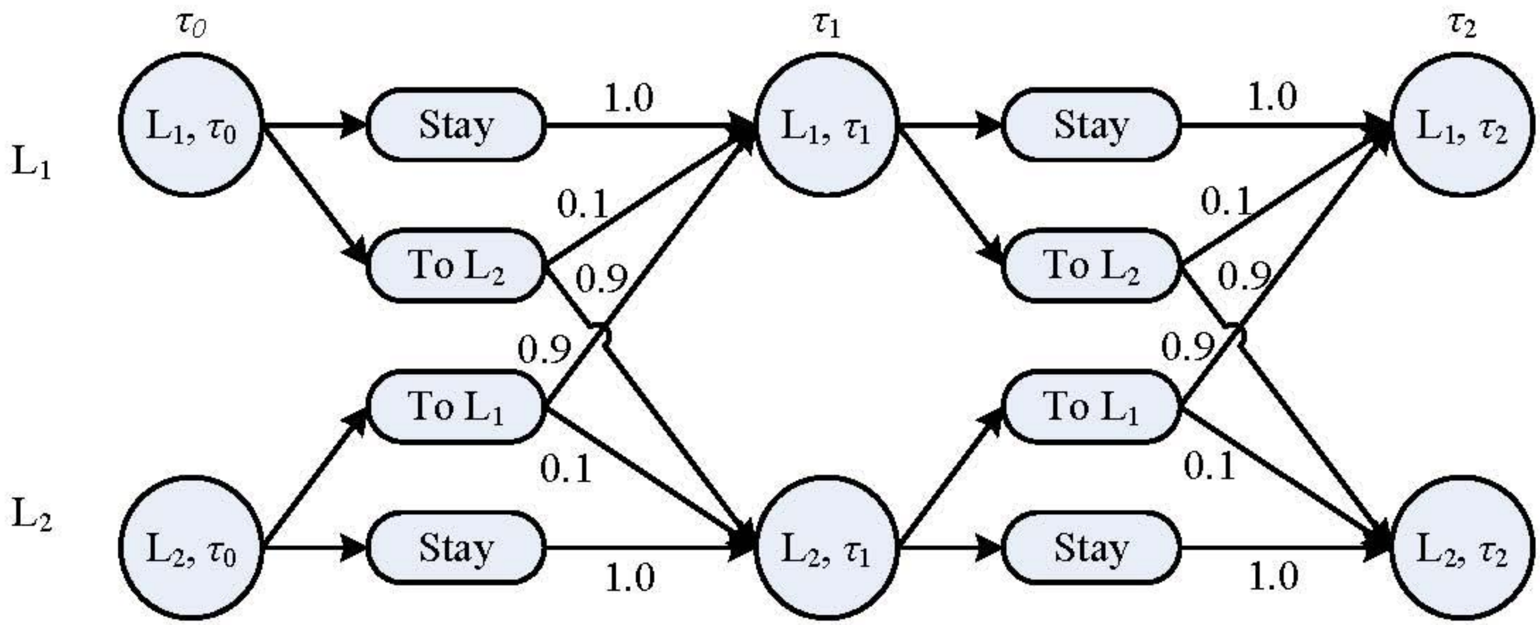
Contributions

- General Stackelberg game model for patrolling with execution uncertainty
  - *Markov Decision Processes model of defender*
  - *Exponential number of defender strategies*
- Efficient algorithm when utility functions are separable
  - *Payoffs decomposed into sum over state transitions*
- Applied to TRUSTS system; deployed at LA Metro

- Planning + Game Theory
  - Execution uncertainty → uncertainty in the environment

Patrolling game with execution uncertainty

- Two-player Bayesian Stackelberg game
  - Leader (defender) has multiple units; an MDP for each unit



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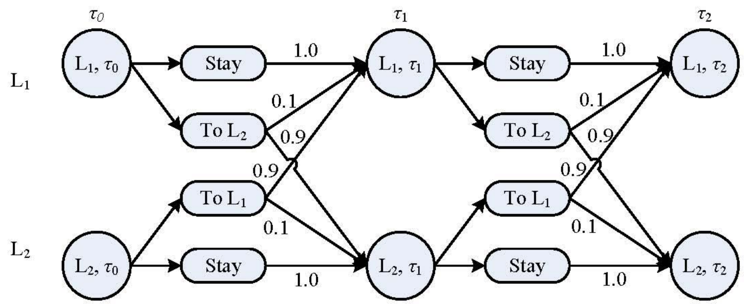
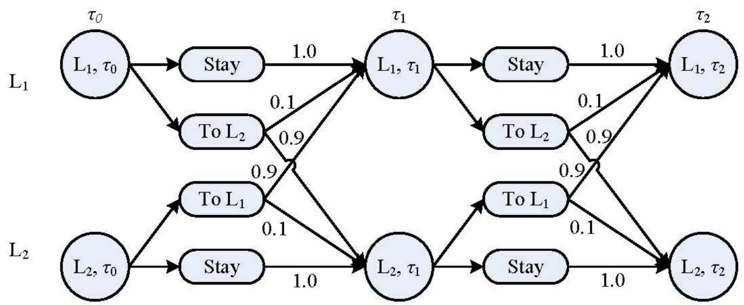


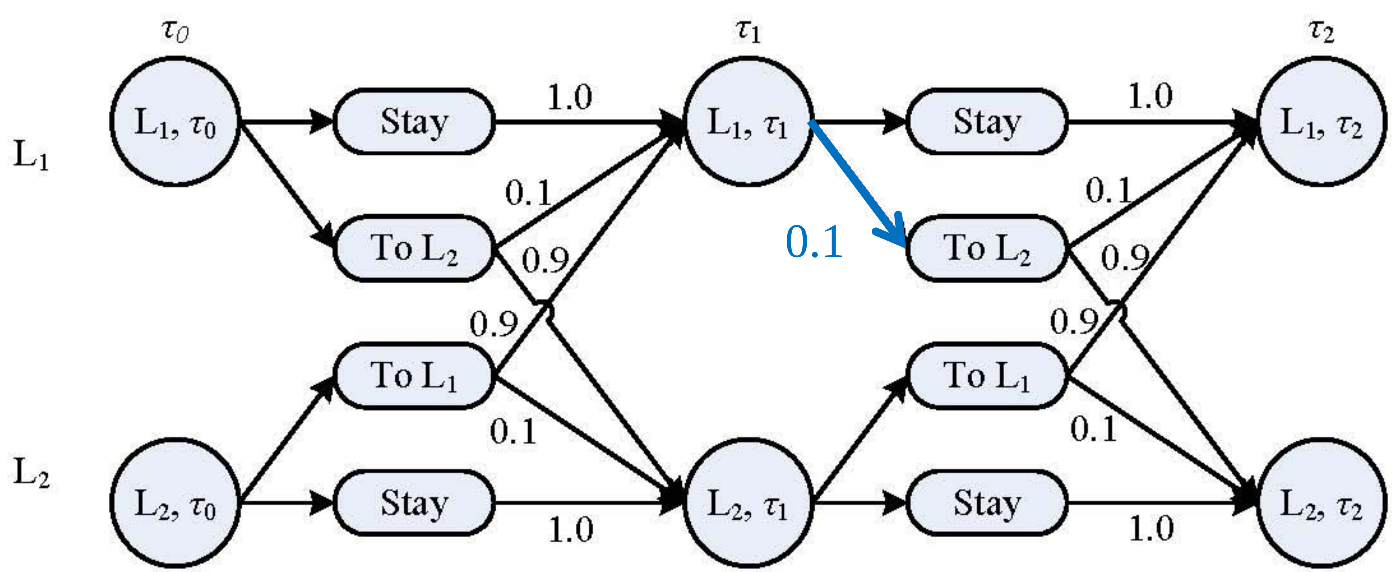


Patrolling game with execution uncertainty

- Two-player Bayesian Stackelberg game
  - Leader (defender) has multiple units; an MDP for each unit
  - Multiple types of attacker
- In general, utility depends on:
  - joint trajectory of defender units (t_1, t_2, …)
  - attacker type λ and action α
- Optimal strategy coupled & non-Markovian

Computation of Stackelberg Equilibrium

- Challenge: exponential # of defender pure strategies
- Compact representation of defender mixed strategies using marginal probabilities $w_i(s,a)$, $x_i(s,a,s')$



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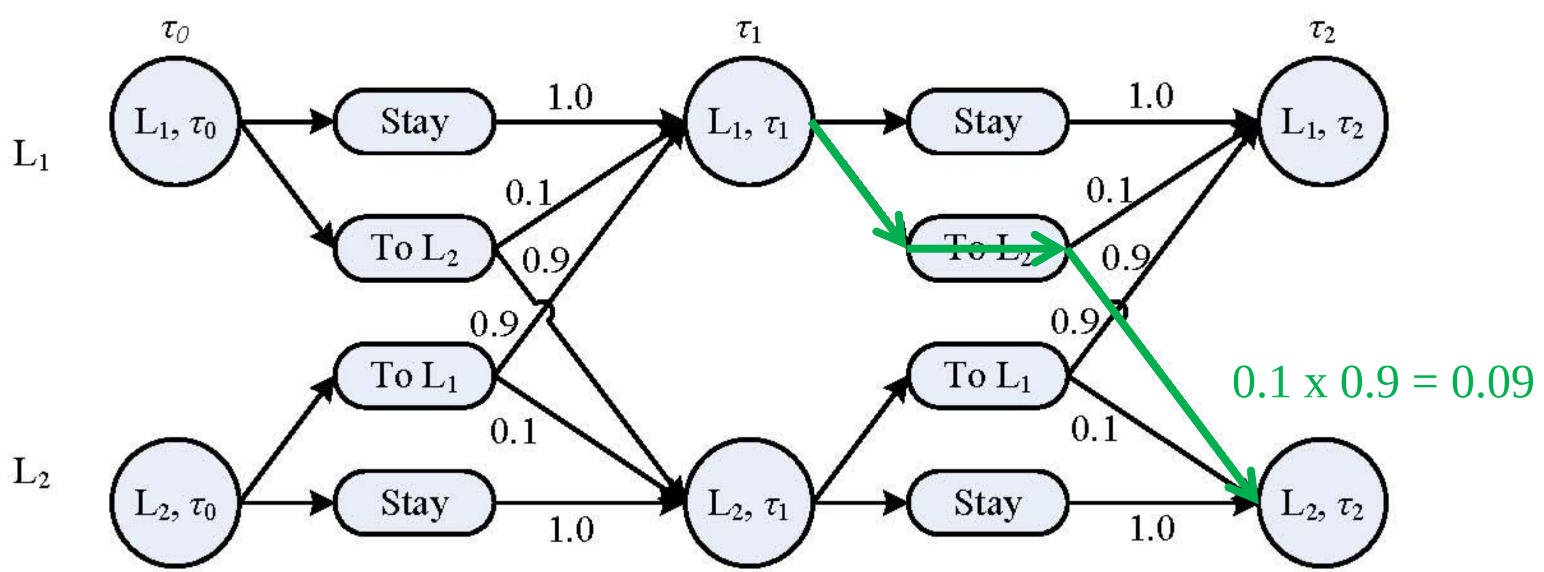

Computation of Stackelberg Equilibrium

- Challenge: exponential # of defender pure strategies
- Compact representation of defender mixed strategies using marginal probabilities $w_i(s,a)$, $x_i(s,a,s')$

$$x_i(s_i, a_i, s_i') = w_i(s_i, a_i) T_i(s_i, a_i, s_i'), \forall s_i, a_i, s_i'$$

$$\sum_{s_i', a_i'} x_i(s_i', a_i', s_i) = \sum_{a_i} w_i(s_i, a_i), \forall s_i$$

$$\sum_{a_i} w_i(s_i^+, a_i) = \sum_{s_i', a_i'} x_i(s_i', a_i', s_i^-) = 1,$$

$$w_i(s_i, a_i) \geq 0, \forall s_i, a_i$$

Computation of Stackelberg Equilibrium

- Can we use this compact representation to solve the game?
  - *Yes, if expected utility can be expressed in terms of* $x_i(s,a,s')$

Computation of Stackelberg Equilibrium

- Can we express expected utility in terms of $x_i(s,a,s')$?
- Yes, if utility functions have separable structure
  - *sum over individual transitions of trajectory*

$$\sum_i \sum_{(s_i,a_i,s_i')\in t_i} U^d(s_i, a_i, s_i', \alpha, \lambda)$$

  - *natural generalization of rewards in MDPs*

LP Formulation for Zero-sum Games

- Size is polynomial using compact representation

$$\max_{\mathbf{w},\mathbf{x},\mathbf{u}} \sum_{\lambda\in\Lambda} p_\lambda u_\lambda + \sum_i \sum_{s_i,a_i,s_i'} x_i(s_i,a_i,s_i')R_i(s_i,a_i,s_i')$$

$$x_i(s_i,a_i,s_i') = w_i(s_i,a_i)T_i(s_i,a_i,s_i'), \forall s_i,a_i,s_i'$$

$$\sum_{s_i',a_i'} x_i(s_i',a_i',s_i) = \sum_{a_i} w_i(s_i,a_i), \forall s_i$$

$$\sum_{a_i} w_i(s_i^+,a_i) = \sum_{s_i',a_i'} x_i(s_i',a_i',s_i^-) = 1,$$

$$w_i(s_i,a_i) \geq 0, \forall s_i,a_i$$

$$u_\lambda \leq \mathbf{x}^T U_\lambda^d \mathbf{e}_\alpha, \forall \lambda \in \Lambda,\ \alpha \in \mathcal{A},$$

LP Formulation for Zero-sum Games

- Size is polynomial using compact representation
- Expected utility linear in $x_i(s,a,s')$

$$\max_{\mathbf{w},\mathbf{x},\mathbf{u}} \sum_{\lambda\in\Lambda} p_\lambda u_\lambda + \sum_i \sum_{s_i,a_i,s_i'} x_i(s_i,a_i,s_i') R_i(s_i,a_i,s_i')$$

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Generating Patrol Schedules

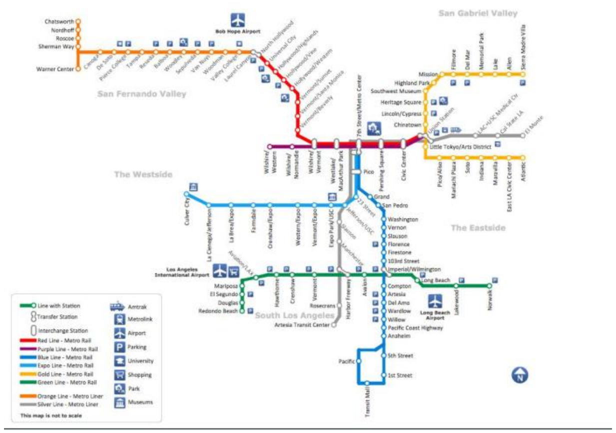
- Calculate decoupled Markov randomized policy
  - State → distribution over actions

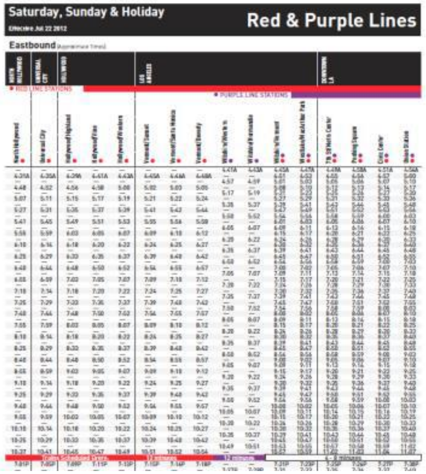
$$\pi_i(s_i, a_i) = \frac{w_i(s_i, a_i)}{\sum_{a_i'} w_i(s_i, a_i')}$$

- Practical deployment:
  - Sample an action from each state
    - deterministic MDP policy; provides contingency plan

Application to LA Metro

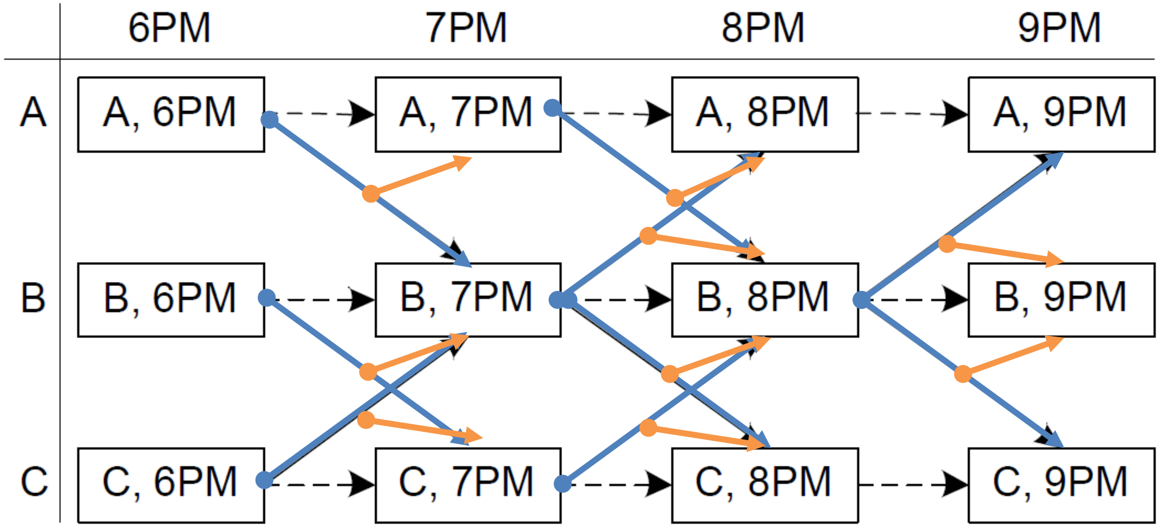
- Red, Blue, Gold, and Green Lines
  - Timetable from http://www.metro.net.
  - Data provided by the LASD.





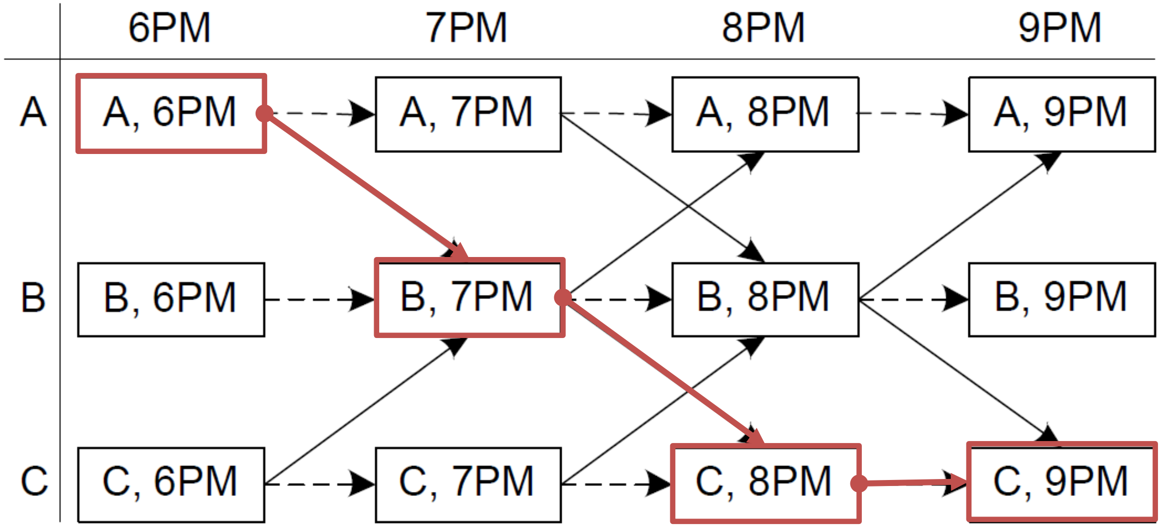
LA Metro: Defender Model

- Defender's MDP for each unit:
  - *Actions: take train or stay at station*
  - *Possibility of delay*



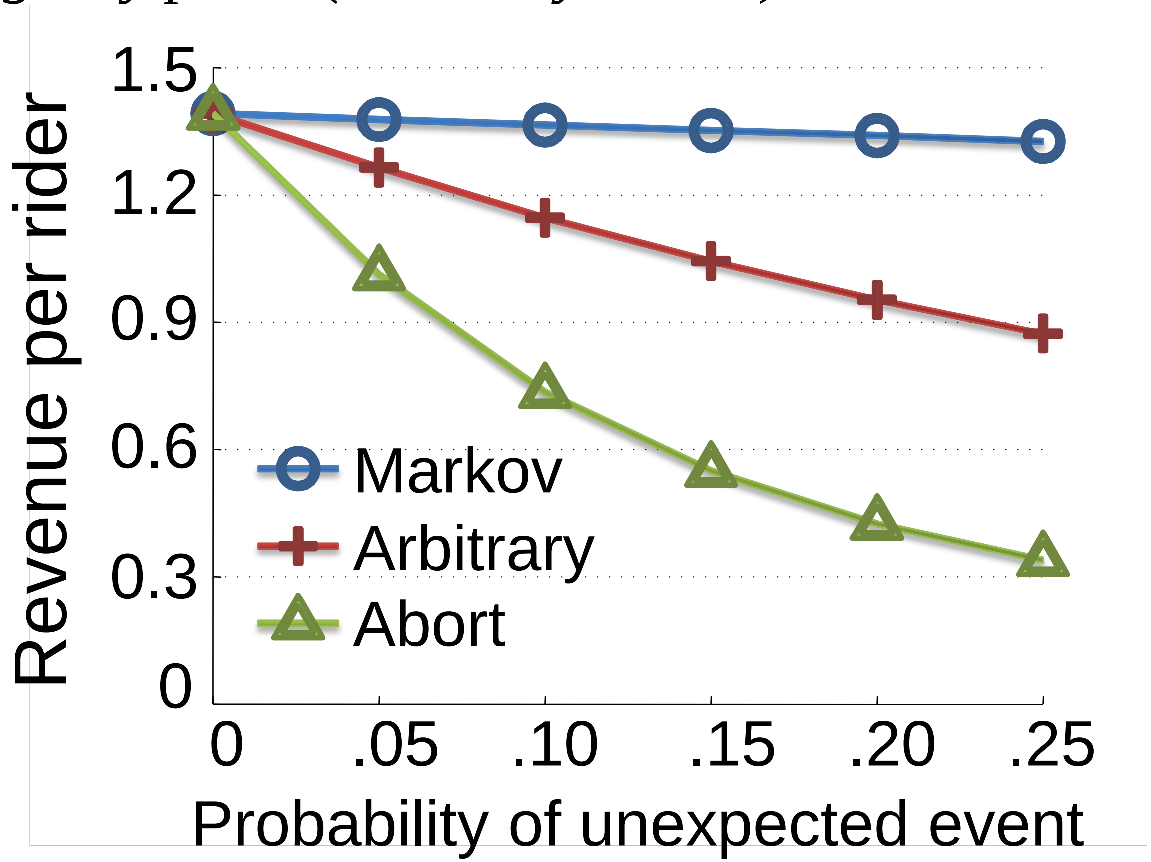
LA Metro: Riders (Potential Fare Evaders)

- Multiple types of riders:
  - *Each type takes fixed route*
  - *Makes a binary decision: buy or not buy the ticket*
- Zero-sum, approximately separable



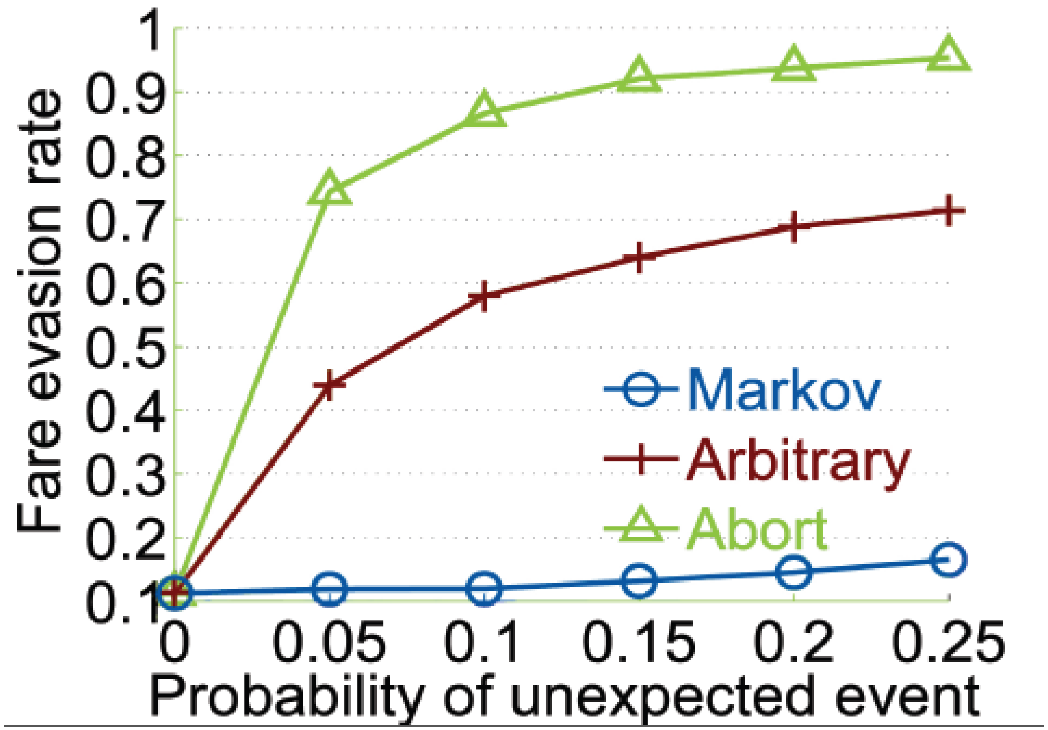
Evaluation

- Revenue per rider with execution uncertainty
  - *Markov strategy (TRUSTSv2) vs. TRUSTSv1 with simple contingency plans (Arbitrary, Abort)*



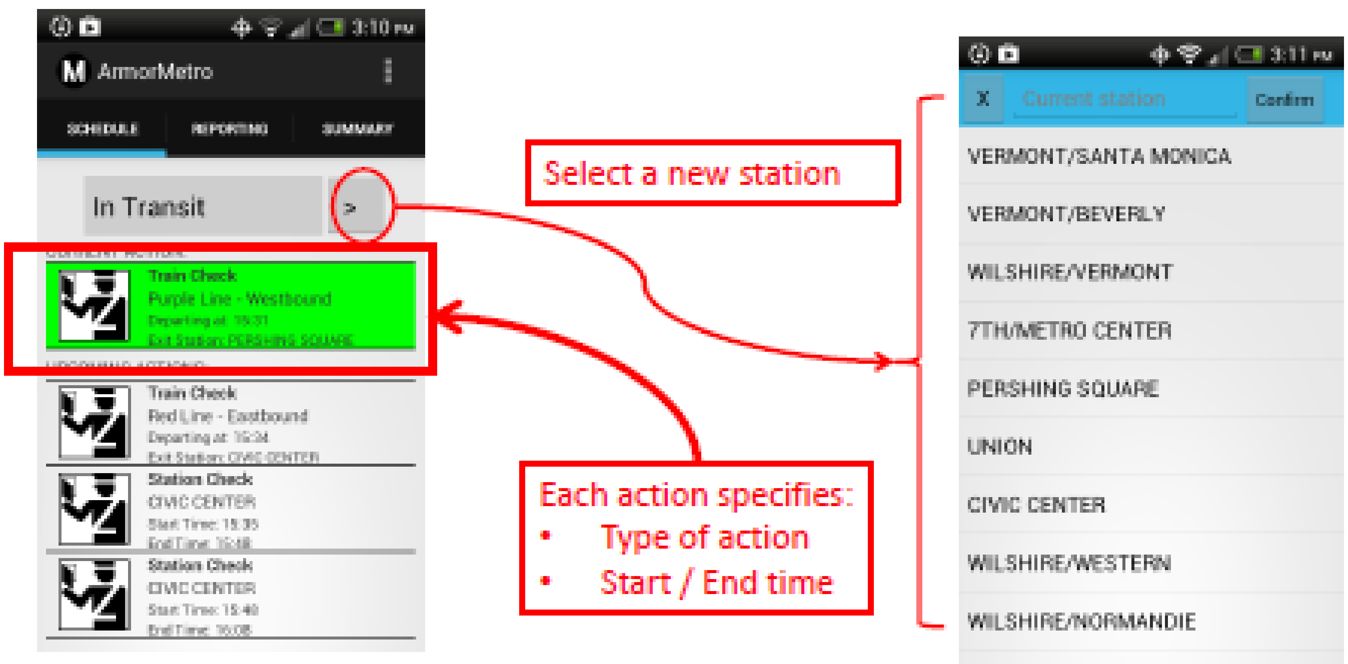
Evaluation

- Fare evasion rate with execution uncertainty
  - *Markov strategy (TRUSTSv2) vs. TRUSTSv1 with simple contingency plans (Arbitrary, Abort)*



TRUSTSv2 Mobile Phone App

- Present patrol strategy (state to action mapping) to officers
- Collect patrol statistics
- Come see our demo! Thursday 10-11am, 3:30-4:30pm



Summary

- Security patrolling with dynamic execution uncertainty
  - Stackelberg game model
  - MDPs to model probabilistic transitions for defender
  - Efficient computation for separable utilities
    - *Compact LP for zero-sum games*
    - *Deployed to the LA Metro domain w/ mobile app*

- Opens door to applications of techniques from planning under uncertainty
  - Avoids intractability of general dynamic games