



# Game-theoretic Randomization for Security Patrolling with Dynamic Execution Uncertainty

---

**Albert Xin Jiang**, Zhengyu Yin, Chao Zhang, Milind Tambe  
University of Southern California  
USA

Sarit Kraus  
Bar-Ilan University  
Israel



# Time-critical Security Patrolling Games

- **Timing** affecting patrol effectiveness



*TRUSTS:*  
Fare inspection in  
LA Metro Rail  
(Yin et al, 2012)



Ferry escort in New York  
(Fang et al, 2013)



*PROTECT:*  
Patrolling Port of Boston  
(Shieh et al, 2012, 2013)

# TRUSTS: Randomized Fare Inspection



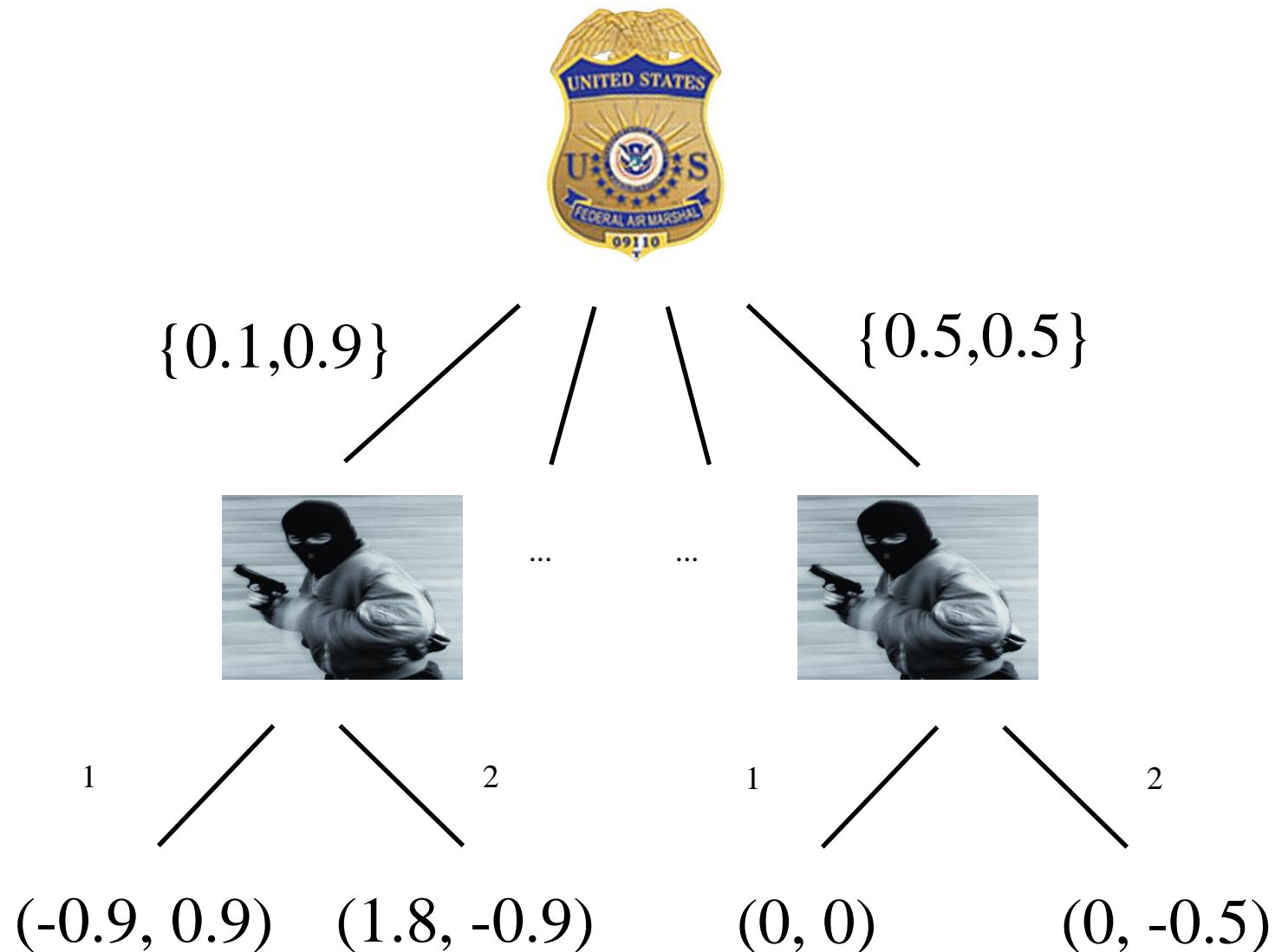
Check fares at “Mission” from 7am to 7:50am  
Go to “Southwest Museum” at 7:50am  
Check fares at “Southwest Museum” from 8am to 9am

- In collaboration with LA Sheriff’s Dept (Yin et al 2012)

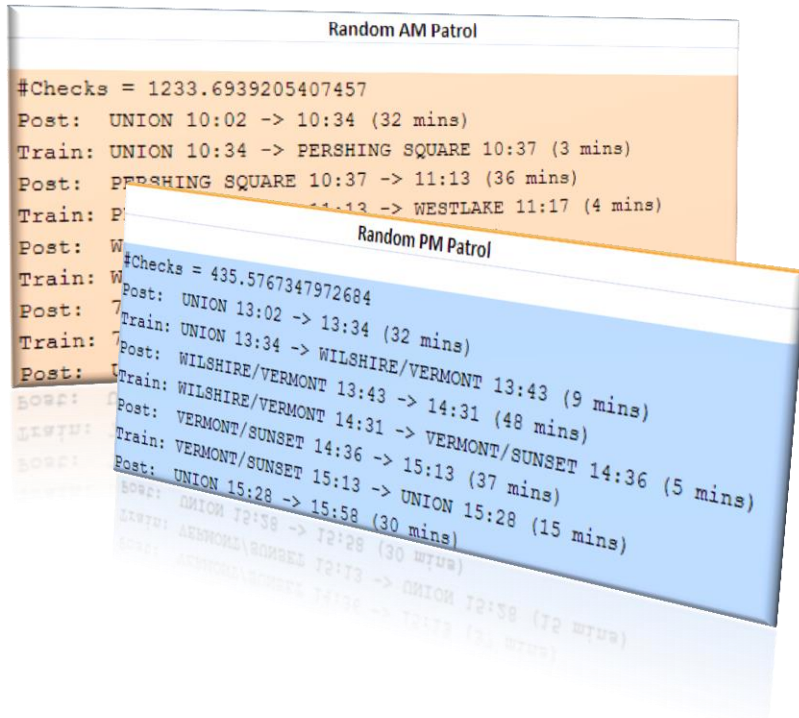
# Stackelberg Equilibrium

Attackers use surveillance in planning attacks

Defender commits to a mixed strategy



# Field tests of TRUSTSv1 (2012)



## Check Statistics

ACTION: POST TRAIN	DESTINATION BEGINNING / TIME	DESTINATION ENDING / TIME	MINUTES	FARE CITES	OTHER CITES	W/A	FARE CKS	ARRESTS	WARRANT CITEOUTS
Post	UNION STA/06:01	UNION STA/06:34	30	155	-	2	155	0	0
TRAIN	UNION STA/06:31	WILSH/VERMONT/06:40	9	0	-	0	25	0	0
Post	WILSH/VERMONT/06:40	WILSH/VERM / 07:16	36	1	-	0	190	0	0
TRAIN	WILSH/VERMONT/07:14	VERMONT/SUNSET/07:41	5	-	0	0	-	11	0
Post	VERMONT/SUNSET/07:41	VERMONT/SUNSET/08:13	32	2	-	4	203	0	0
Post	VERMONT/SUNSET/07:35	VERMONT/SUNSET/08:13	38	2	-	4	203	0	0
TRAIN	VERMONT/SUNSET/08:13	UNION STA/08:28	15	0	-	0	50	0	0
Post	UNION STA/08:28	UNION STA/09:00	30	1	-	1	94	0	0
TOTALS =				161	-	5	717	0	0
TRIED THIS SCHEDULE WITH 4 SAs (2-2MAN TEAMS) TO SEE IF THE NUMBERS INCREASED									

## Remarks

Feedback: officer often deviate from schedules

- *Felony arrest*
- *Called to deal with emergencies*

# Problem: Dynamic Execution Uncertainty

- **Execution uncertainty** can affect the defender units' ability to carry out their planned schedules in later time steps
- Want **robust** patrol schedules with **contingency plans**
- Related work on uncertainty in security games
  - *either only applicable to static domains* (Yin et al 2011; 2012)
  - *or consider general dynamic game formulations (NP-hard)*
    - (Letchford et al 2010; 2012; Vorobeychik et al 2012)

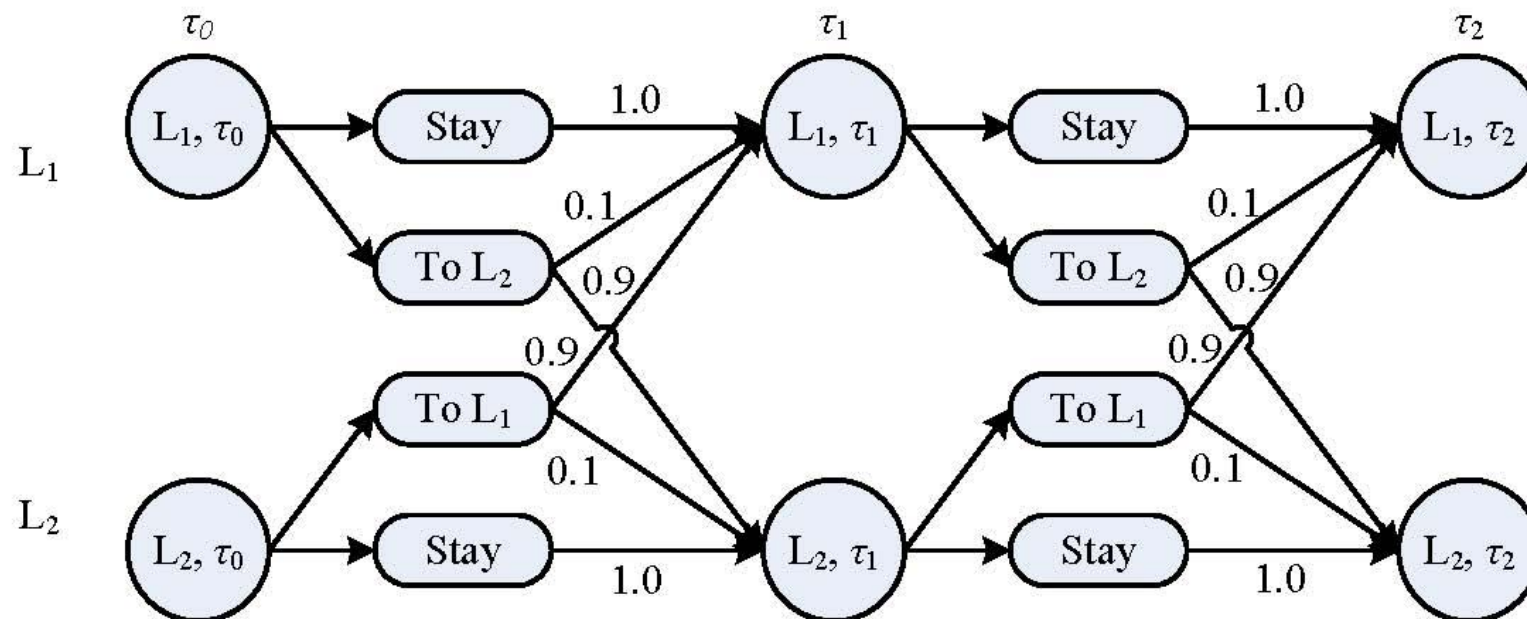
# Contributions

---

- General Stackelberg game model for patrolling with execution uncertainty
  - *Markov Decision Processes model of defender*
  - *Exponential number of defender strategies*
- Efficient algorithm when utility functions are **separable**
  - *Payoffs decomposed into sum over state transitions*
- Applied to TRUSTS system; deployed at LA Metro
- **Planning + Game Theory**
  - Execution uncertainty → uncertainty in the environment

# Patrolling game with execution uncertainty

- Two-player Bayesian Stackelberg game
  - Leader (defender) has multiple units; an **MDP** for each unit

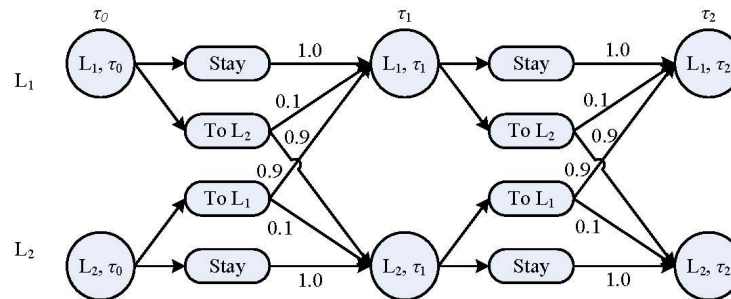
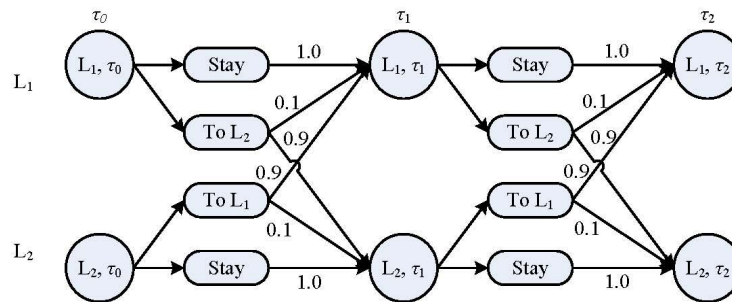




# Patrolling game with execution uncertainty

## ■ Two-player Bayesian Stackelberg game

- Leader (defender) has multiple units; an **MDP** for each unit

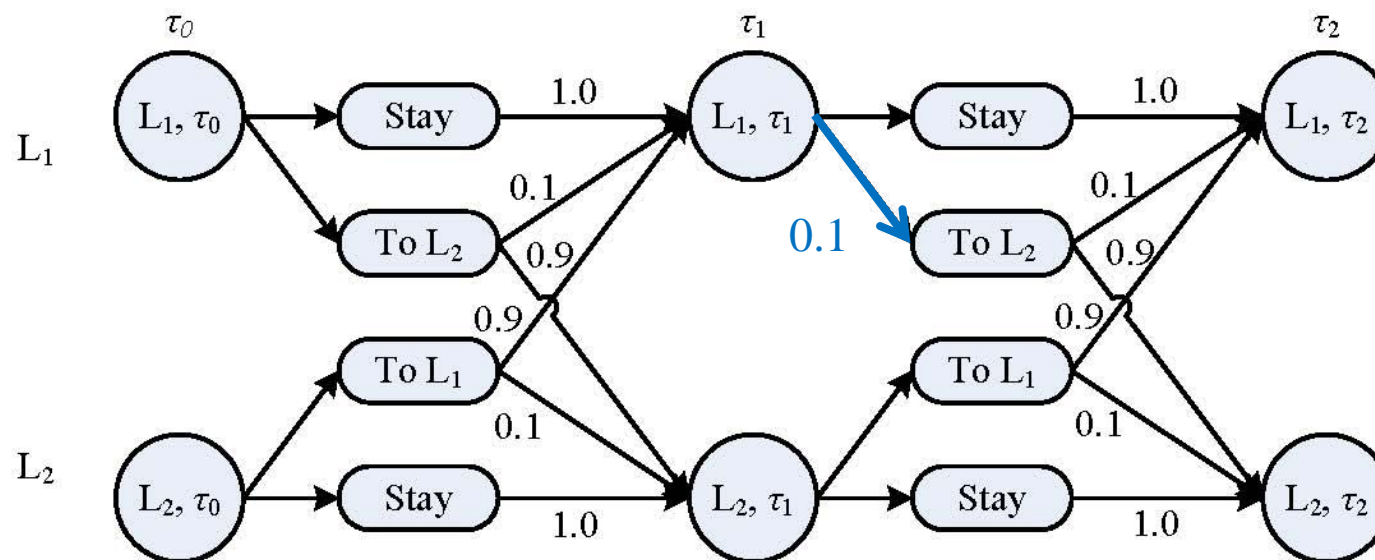


# Patrolling game with execution uncertainty

- Two-player Bayesian Stackelberg game
  - Leader (defender) has multiple units; an **MDP** for each unit
  - Multiple types of attacker
- In general, utility depends on:
  - joint trajectory of defender units  $(t_1, t_2, \dots)$
  - attacker type  $\lambda$  and action  $\alpha$
- Optimal strategy **coupled & non-Markovian**

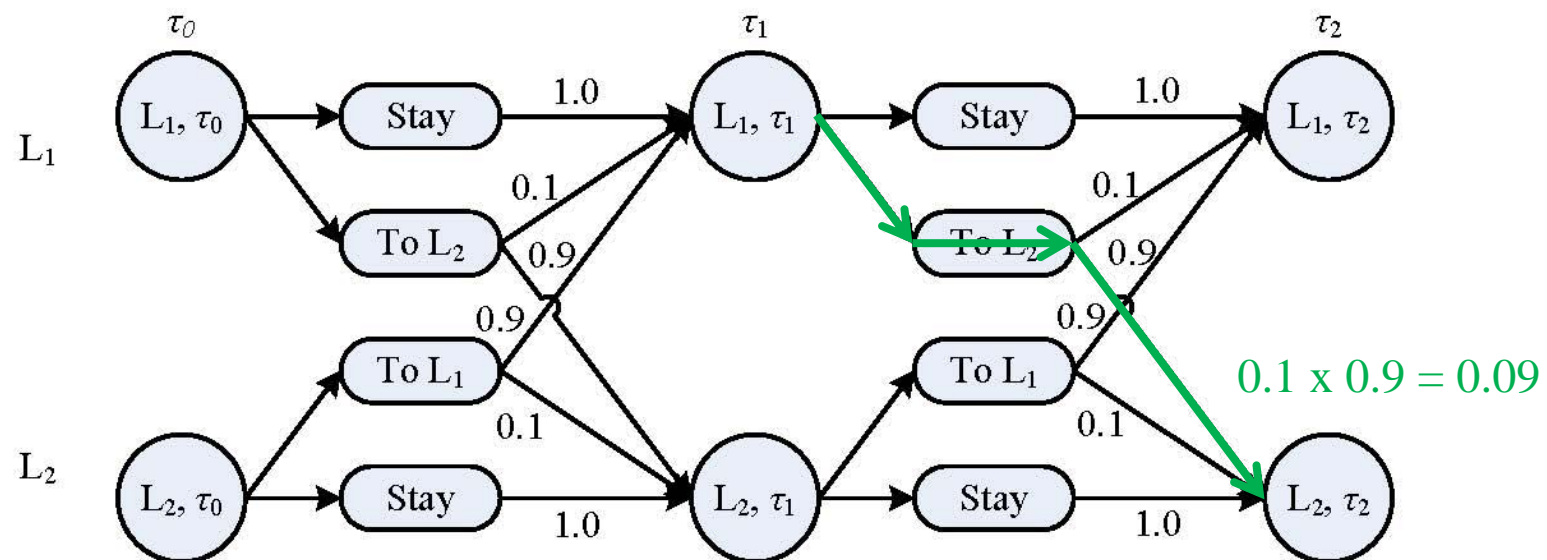
# Computation of Stackelberg Equilibrium

- Challenge: **exponential** # of defender pure strategies
- Compact representation of defender **mixed strategies** using marginal probabilities  $w_i(s,a)$ ,  $x_i(s,a,s')$



# Computation of Stackelberg Equilibrium

- Challenge: **exponential** # of defender pure strategies
- Compact representation of defender **mixed strategies** using marginal probabilities  $w_i(s,a)$ ,  $x_i(s,a,s')$



# Computation of Stackelberg Equilibrium

- Challenge: **exponential** # of defender pure strategies
- Compact representation of defender **mixed strategies** using marginal probabilities  $w_i(s, a)$ ,  $x_i(s, a, s')$

$$x_i(s_i, a_i, s'_i) = w_i(s_i, a_i)T_i(s_i, a_i, s'_i), \forall s_i, a_i, s'_i$$

$$\sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i) = \sum_{a_i} w_i(s_i, a_i), \forall s_i$$

$$\sum_{a_i} w_i(s_i^+, a_i) = \sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i^-) = 1,$$

$$w_i(s_i, a_i) \geq 0, \forall s_i, a_i$$

# Computation of Stackelberg Equilibrium

---

- Can we use this compact representation to solve the game?
  - *Yes, if expected utility can be expressed in terms of  $x_i(s, a, s')$*

# Computation of Stackelberg Equilibrium

- Can we express expected utility in terms of  $x_i(s, a, s')$ ?
- Yes, if utility functions have **separable** structure
  - *sum over individual transitions of trajectory*

$$\sum_i \sum_{(s_i, a_i, s'_i) \in t_i} U^d(s_i, a_i, s'_i, \alpha, \lambda)$$

- *natural generalization of rewards in MDPs*

# LP Formulation for Zero-sum Games

- Size is polynomial using compact representation

$$\max_{\mathbf{w}, \mathbf{x}, \mathbf{u}} \sum_{\lambda \in \Lambda} p_{\lambda} u_{\lambda} + \sum_i \sum_{s_i, a_i, s'_i} x_i(s_i, a_i, s'_i) R_i(s_i, a_i, s'_i)$$

$$x_i(s_i, a_i, s'_i) = w_i(s_i, a_i) T_i(s_i, a_i, s'_i), \forall s_i, a_i, s'_i$$

$$\sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i) = \sum_{a_i} w_i(s_i, a_i), \forall s_i$$

$$\sum_{a_i} w_i(s_i^+, a_i) = \sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i^-) = 1,$$

$$w_i(s_i, a_i) \geq 0, \forall s_i, a_i$$

$$u_{\lambda} \leq \mathbf{x}^T U_{\lambda}^d \mathbf{e}_{\alpha}, \forall \lambda \in \Lambda, \alpha \in \mathcal{A},$$



# LP Formulation for Zero-sum Games

- Size is polynomial using compact representation
- Expected utility linear in  $x_i(s, a, s')$

$$\max_{\mathbf{w}, \mathbf{x}, \mathbf{u}} \sum_{\lambda \in \Lambda} p_\lambda u_\lambda + \sum_i \sum_{s_i, a_i, s'_i} x_i(s_i, a_i, s'_i) R_i(s_i, a_i, s'_i)$$

$$x_i(s_i, a_i, s'_i) = w_i(s_i, a_i) T_i(s_i, a_i, s'_i), \forall s_i, a_i, s'_i$$

$$\sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i) = \sum_{a_i} w_i(s_i, a_i), \forall s_i$$

$$\sum_{a_i} w_i(s_i^+, a_i) = \sum_{s'_i, a'_i} x_i(s'_i, a'_i, s_i^-) = 1,$$

$$w_i(s_i, a_i) \geq 0, \forall s_i, a_i$$

$$u_\lambda \leq \mathbf{x}^T U_\lambda^d \mathbf{e}_\alpha, \forall \lambda \in \Lambda, \alpha \in \mathcal{A},$$

# Generating Patrol Schedules

- Calculate **decoupled** Markov randomized policy

- State  $\rightarrow$  distribution over actions

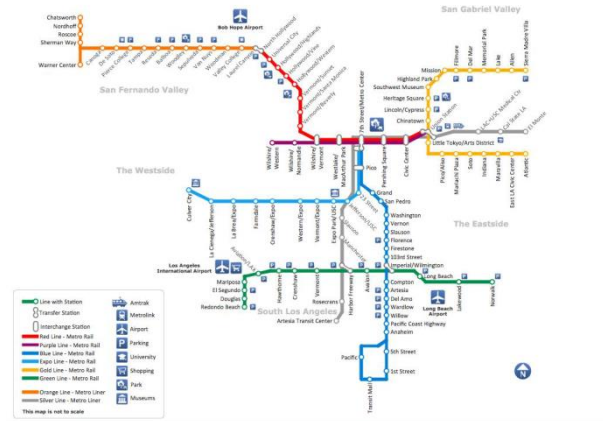
$$\pi_i(s_i, a_i) = \frac{w_i(s_i, a_i)}{\sum_{a'_i} w_i(s_i, a'_i)}$$

- Practical deployment:

- Sample an action from each state
  - deterministic MDP policy; provides contingency plan

# Application to LA Metro

- Red, Blue, Gold, and Green Lines
  - Timetable from <http://www.metro.net>.
  - Data provided by the LASD.



**Saturday, Sunday & Holiday**  
Effective Jul 22 2012

**Red & Purple Lines**

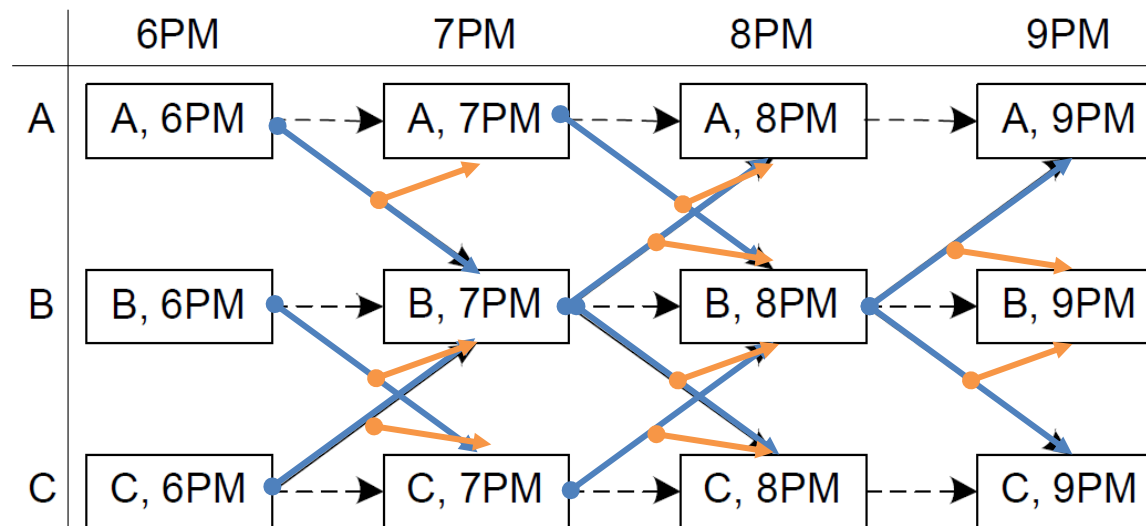
Eastbound (Approximate times)

Line	Station	10:00 AM	10:30 AM	11:00 AM	11:30 AM	12:00 PM	12:30 PM	1:00 PM	1:30 PM	2:00 PM	2:30 PM	3:00 PM	3:30 PM	4:00 PM	4:30 PM	5:00 PM	5:30 PM	6:00 PM
Red Line	Alhambra																	
	San Gabriel																	
Purple Line	Union Station																	
	Harbor Gateway																	

The table continues with detailed arrival and departure times for various stations along the Red and Purple lines, including Union Station, Harbor Gateway, and others. It also includes information about Metro Rail, Metro Bus, and Metro Bike Share services.

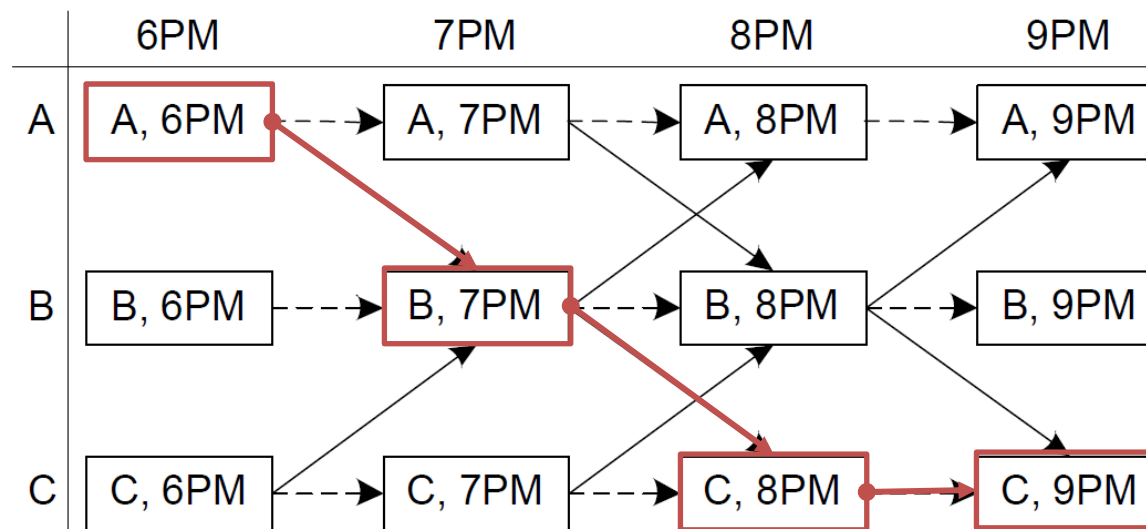
# LA Metro: Defender Model

- Defender's MDP for each unit:
  - *Actions: take train or stay at station*
  - *Possibility of delay*



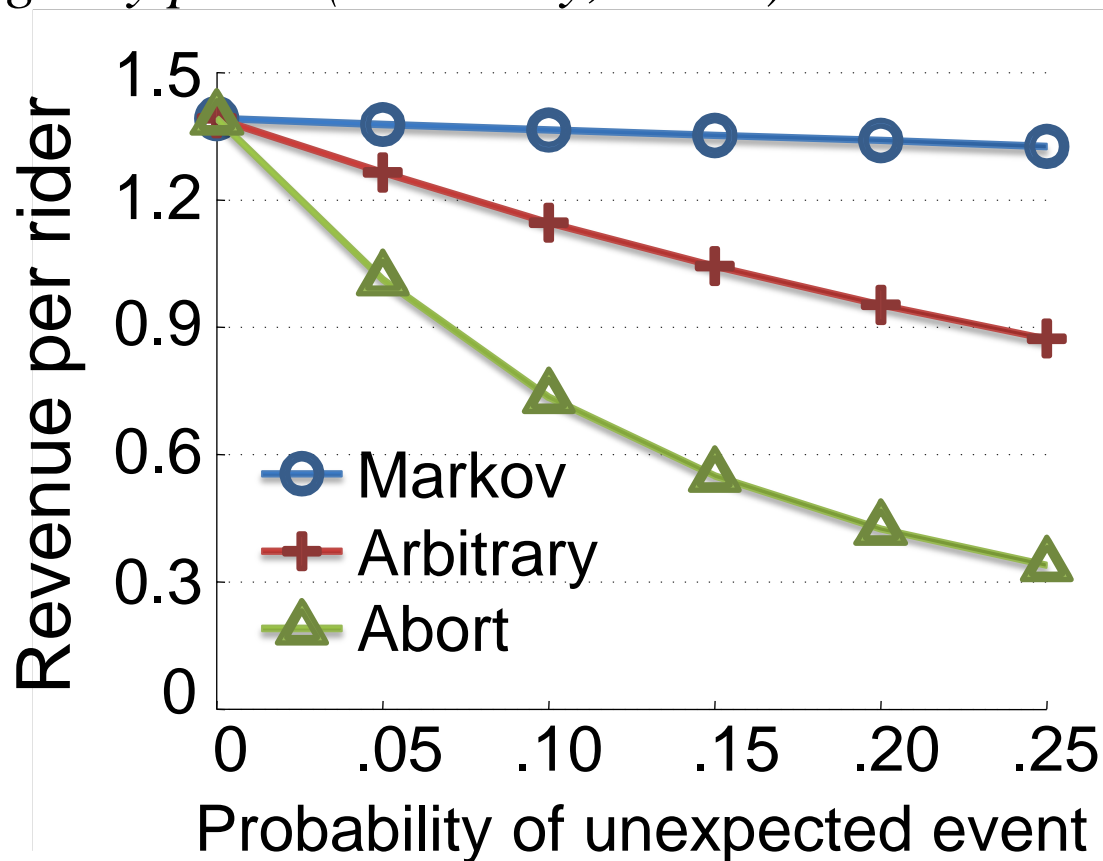
# LA Metro: Riders (Potential Fare Evaders)

- Multiple types of riders:
  - *Each type takes fixed route*
  - *Makes a binary decision: buy or not buy the ticket*
- Zero-sum, approximately separable



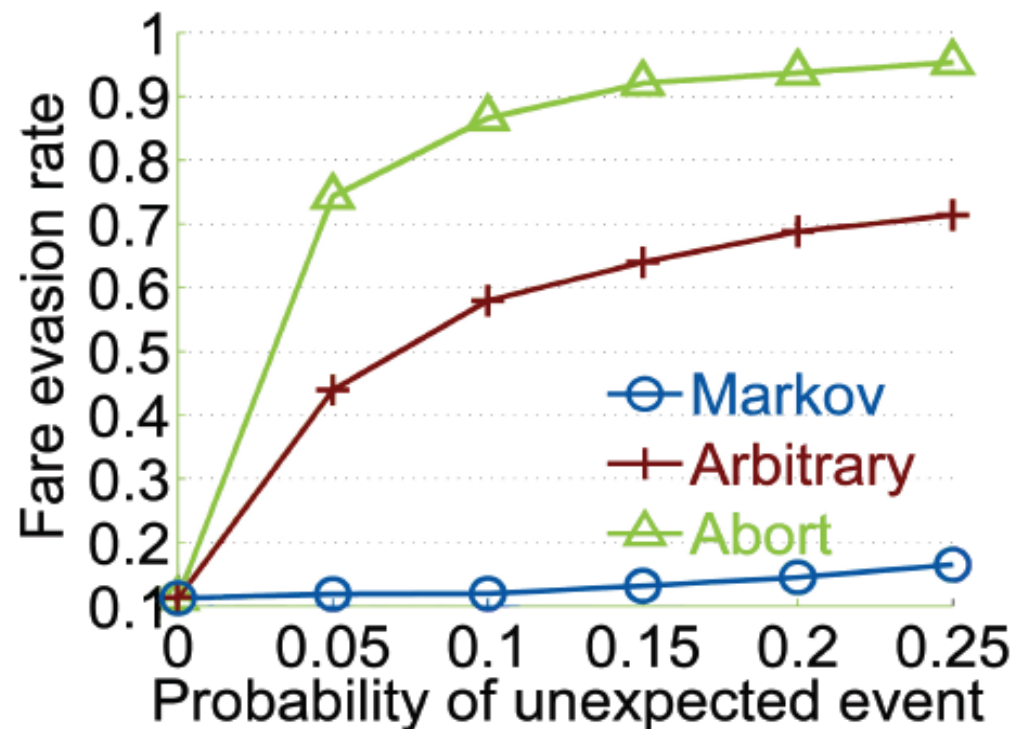
# Evaluation

- Revenue per rider with execution uncertainty
  - *Markov strategy (TRUSTSv2) vs. TRUSTSv1 with simple contingency plans (Arbitrary, Abort)*



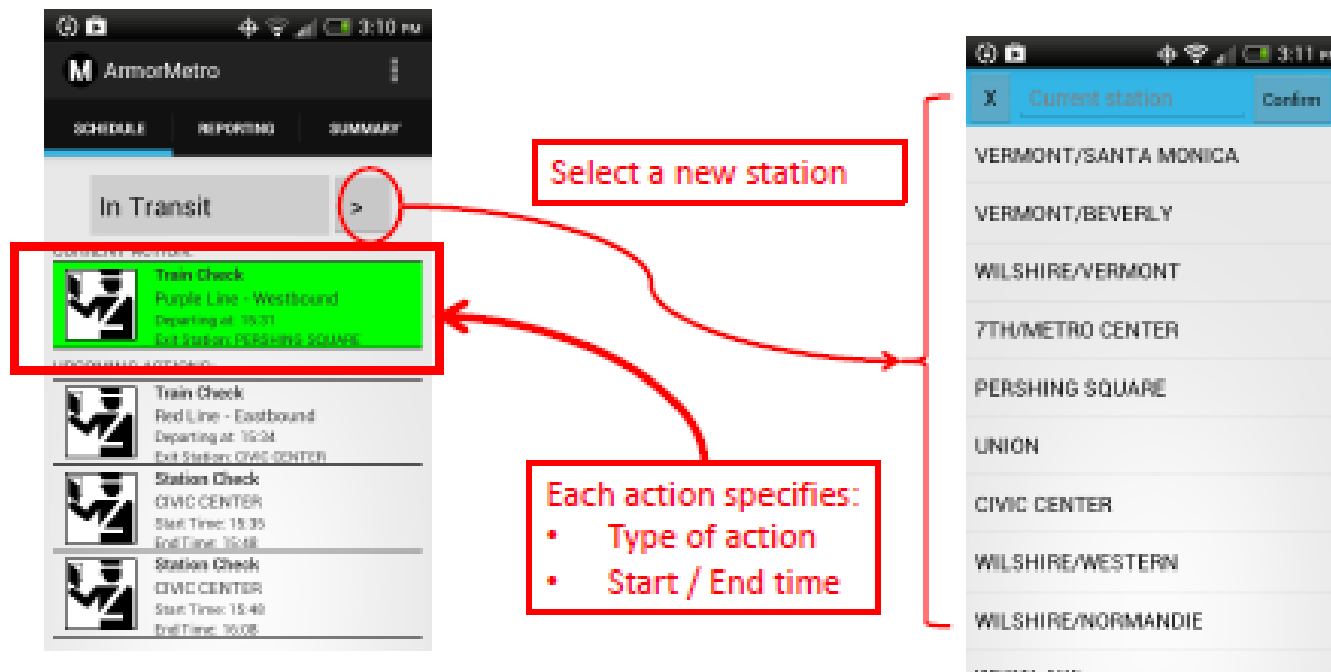
# Evaluation

- Fare evasion rate with execution uncertainty
  - *Markov strategy (TRUSTSv2) vs. TRUSTSv1 with simple contingency plans (Arbitrary, Abort)*



# TRUSTSv2 Mobile Phone App

- Present patrol strategy (state to action mapping) to officers
- Collect patrol statistics
- Come see our demo! Thursday 10-11am, 3:30-4:30pm





# Summary

---

- Security patrolling with dynamic execution uncertainty
  - Stackelberg game model
  - MDPs to model probabilistic transitions for defender
  - Efficient computation for **separable** utilities
    - *Compact LP for zero-sum games*
    - *Deployed to the LA Metro domain w/ mobile app*
- Opens door to applications of techniques from planning under uncertainty
  - Avoids intractability of general dynamic games