Appendix for: Monotonic Maximin: A Robust Stackelberg Solution Against Boundedly Rational Followers

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A Proof of Proposition 7

Proof. Given $\boldsymbol{x}$, Denote by U_i^l and U_i^f the expected utilities of the leader and the follower respectively if the follower chooses action i. We have $U_i^l = x^T A e_i$ and $U_i^f = x^T B e_i$.

Let $(\boldsymbol{x}, \boldsymbol{\lambda}, t)$ be an optimal solution of the LP for F^t. Then it is straightforward to verify that for each $F^c \in \mathcal{F}(\mathcal{E}^t)$, $\boldsymbol{x}^T B F^c \geq 0$.

We first consider the simple case that there exists only 2 targets (i, j) such that $U_i^f = U_j^f$; suppose $F^t_{(i,j)}$ and $F^t_{(j,i)}$ are the two columns of F^t corresponding to (i, j) and (j, i), respectively. Suppose $\lambda_{(i,j)} \geq \lambda_{(j,i)}$ w.l.o.g. Choose F^c such that $F^c_{(i,j)} = F^t_{(i,j)}$ and $F^c_{(j,i)} = 0$. Choose $\hat{\boldsymbol{\lambda}}$ such that $\hat{\lambda}_k = \lambda_k, \forall k \neq (i, j)$ and $\hat{\lambda}_{(i,j)} = \lambda_{(i,j)} - \lambda_{(j,i)}$. Then it is straightforward to verify that $F^c \hat{\boldsymbol{\lambda}} = F^t \boldsymbol{\lambda}$, and thus $(\boldsymbol{x}, \hat{\boldsymbol{\lambda}}, t)$ is a feasible solution of the LP for F^c which implies $V_{F^t} \leq V_{F^c}$.

For the general case, we construct a corresponding "strict" order matrix F^c in a similar way: for every pair of targets (i, j) such that $(i, j), (j, i) \in \mathcal{E}^t$, choose $F^c_{(i,j)}, F^c_{(j,i)}, \lambda_{(i,j)}, \lambda_{(j,i)}$ similarly as the previous simple case. By a similar argument, $(\boldsymbol{x}, \hat{\boldsymbol{\lambda}}, t)$ is a feasible solution of the LP for F^c.

In other words, given an optimal solution $(\boldsymbol{x}, \boldsymbol{\lambda}, t)$ of the LP for F^t, we can always find a F^c and a feasible solution of the corresponding LP with the same objective value t. Therefore, $V_{F^t} \leq \max_{F^c \in \mathcal{F}(\mathcal{E}^t)} \{V_{F^c}\}$.