

HW1 Truth Table and Logical Equivalence  
(Due at the beginning of class on Friday Feb. 2)

Use your own paper, type your answers, and show work as much as possible.

**Basic Prove Technology**

1. (8 points) Specify the type of the following problem (*Find* or *Proof*) and use *Working Backward* to devise a solution plan.  
Given a 4 quart pail and a 9 quart pail, obtain 6 quarts of water in the 9 quart pail using these two pails. You can fill or empty the pails and you can have as much water as you want.

**Basic Connectives and Truth Table**

2. (2 points each, total 6 points) Determine the truth value of each of the following implications.
  - a. If  $3+4 = 12$ , then  $3+2 = 6$ .
  - b. If  $3+3 = 6$ , then  $3+4 = 9$ .
  - c. If Thomas Jefferson was the third president of the United States, then  $2+3 = 5$ .
3. (3 points each, total 24 points) Construct a truth table for each of the following compound statements. Which of the compound statements are tautologies. (Here  $p$ ,  $q$ ,  $r$  denote primitive statements)
  - a.  $(p \vee \neg q) \rightarrow \neg p$
  - b.  $p \rightarrow (q \rightarrow r)$
  - c.  $(p \rightarrow q) \rightarrow r$
  - d.  $(p \rightarrow q) \rightarrow (q \rightarrow p)$
  - e.  $[p \wedge (p \rightarrow q)] \rightarrow q$
  - f.  $(p \wedge q) \rightarrow p$
  - g.  $q \leftrightarrow (p \vee q)$
  - h.  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
4. (8 points) Verify that  $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$  is a tautology.
5. (2 points each, total 8 points) At the start of a program (written in pseudocode) the integer variable  $n$  is assigned the value 7. Determine the value of  $n$  after each of the following *successive* statements is encountered during the execution of this program. [Here the value of  $n$  following the execution of the statement in part (a) becomes the value of  $n$  for the statement in part (b), and so on, through the statement in part (d). For positive integers  $a$ ,  $b$ ,  $\lfloor \frac{a}{b} \rfloor$  returns the integer part of the quotient – for example,  $\lfloor \frac{6}{2} \rfloor = 3$ ,  $\lfloor \frac{7}{2} \rfloor = 3$ ,  $\lfloor \frac{2}{5} \rfloor = 0$ , and  $\lfloor \frac{8}{3} \rfloor = 2$ .
  - a. **if**  $n > 5$  **then**  $n := n + 2$
  - b. **if**  $((n + 2 = 8)$  **or**  $(n - 3 = 6))$  **then**  $n := 2 * n + 1$
  - c. **if**  $((n - 3 = 16)$  **and**  $(\lfloor \frac{n}{6} \rfloor = 1))$  **then**  $n := n + 3$
  - d. **if**  $((n \neq 21)$  **and**  $(n - 7 = 15))$  **then**  $n := n - 4$

### Logical Equivalence: The laws of Logic

6. (6 points each, total 12 points) Use logical equivalence and the substitution rules to verify that each of the following is a tautology. (Here,  $p$ ,  $q$ , and  $r$  are primitive statements).
- $[p \vee (q \wedge r)] \vee \neg[p \vee (q \wedge r)]$
  - $[(p \vee q) \rightarrow r] \leftrightarrow [\neg r \rightarrow \neg(p \vee q)]$
7. (5 points each, total 10 points) Negate and express each of the following statements in smooth English.
- Kelsey will get a good education if she puts her studies before her interest in cheerleading.
  - Norma is doing her homework, and Karen is practicing her piano lessons.
8. (8 points each, total 16 points) Provide the steps and reasons to establish the following logical equivalences. Also write the duals of these logical equivalences.
- $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$
  - $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$
9. (4 points each, total 8 points) Write the converse, inverse, and contrapositive of each of the following implications. For each implication, determine its truth value as well as the truth values of its corresponding converse, inverse, and contrapositive.
- If  $0 + 0 = 0$ , then  $1 + 1 = 1$ .
  - If  $-1 < 3$  and  $3+7 = 10$ , then  $\sin\left(\frac{3\pi}{2}\right) = -1$ .