CSCI1323 HW2 Derivation Rules and Quantifiers
(Due at the beginning of the class in Friday, Feb. 15)

Use your own paper, type your answers and show work as much as possible.

Derivation Rules

1. (2 points each, total 10 points)Consider each of the following arguments. If the argument is valid, identify the rule of inference that establishes its validity. If not, indicate the error is due to which fallacy.
   a) Andrea can program in C++, and she can program in Java.
   Therefore Andrea can program in C++.
   b) A sufficient condition for Bubbles to win the golf tournament is that her opponent Meg not sink a birdie on the last hole.
   Bubbles won the golf tournament.
   Therefore Bubbles' opponent Meg did not sink a birdie on the last hole.
   c) If Ron's computer program is correct, then he'll be able to complete his computer science assignment in at most two hours.
   I takes Ron over two hours to complete his computer science assignment.
   Therefore Ron's computer program is not correct.
   d) Eileen's car keys are in her purse, or they are on the kitchen table.
   Eileen's car keys are not on the kitchen table.
   Therefore Eileen's car keys are in her purse.
   e) If interest rates fall, then the stock market will rise.
   Interest rates are not falling.
   There fore the stock market will not rise.

2. (3 points each, total 24 points) Establish the validity of the following arguments.
   a) $[(p \wedge \neg q) \wedge r] \rightarrow [(p \wedge r) \vee q]$
   b) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$
   c) $p \rightarrow q$
   $\neg q$
   $\underline{\neg r\quad}$
   $\neg(p \vee r)$
   d) $p \rightarrow q$
   $r \rightarrow \neg q$
   $\underline{r\qquad\quad}$
   $\neg p$
   e) $p \rightarrow (q \rightarrow r)$
   $\neg q \rightarrow \neg p$
   $\underline{p\qquad\quad}$
   r
   f) $p \wedge q$
   $p \rightarrow (r \wedge q)$
   $r \rightarrow (s \vee t)$
   $\underline{\neg s\qquad\quad}$

t

g) $p \rightarrow (q \rightarrow r)$
   $p \vee s$
   $t \rightarrow q$
   $\underline{\neg s \quad\quad}$
   $\neg r \rightarrow \neg t$

h) $p \vee q$
   $\neg p \vee r$
   $\underline{\neg r \quad}$
   q

3. (6 points each, total 18 points) Write each of the following arguments in symbolic form. Then establish the validity of the argument by using **Proof by Contradiction**, or give a counter-example to show that it is invalid.
   a) If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the supervisor's position or she did not work hard.
   b) If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.
   c) If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 80°F, there is no chance for rain. Today the temperature is 85°F and Lois is wearing her red headband. Therefore (sometime today) Lois will mow her lawn.

Quantifiers

4. (8 points each, total 32 points) For the universe of all integers, let $p(x)$, $q(x)$, $r(x)$, $s(x)$, and $t(x)$ be the following open statements.

   $p(x)$: $x>0$
   $q(x)$: x is even
   $r(x)$: x is a perfect square
   $s(x)$: x is (exactly) divisible by 4
   $t(x)$: x is (exactly) divisible by 5

   a) Write the following statements in symbolic form.
      i) At least one integer is even.
      ii) There exists a positive integer that is even.
      iii) If x is even, then x is not divisible by 5.
      iv) No even integer is divisible by 5.
      v) There exists an even integer divisible by 5.
      vi) If x is even and x is perfect square, then x is divisible by 4.

b) Determine whether each of the six statements in part (a) is true or false. For each false statement, provide a counterexample.
c) Express each of the following symbolic representations in words.
   i) $\forall x[r(x) \rightarrow p(x)]$
   ii) $\forall x[s(x) \rightarrow q(x)]$
   iii) $\forall x[s(x) \rightarrow \neg t(x)]$
   iv) $\exists x[s(x) \wedge \neg r(x)]$
d) Provide a counterexample for each false statement in part (c).

5. (4 points each, total 16 points) Negate and simplify each of the following.
   a) $\exists x[p(x) \vee q(x)]$
   b) $\forall x[p(x) \wedge \neg q(x)]$
   c) $\forall x[p(x) \rightarrow q(x)]$
   d) $\exists x[p(x) \vee q(x)) \rightarrow r(x)]$