# CSCI1323 Quiz 2 Key

Prove the following open statement by mathematic induction.

$$S(n):\ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof:

(a) Basis step. Prove S(1) is T, i.e. when n=1, $\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2\times 1+1)}{6}$.

Proof: $\sum_{i=1}^{1} i^2 = 1^2 = 1$,

$$\frac{n(n+1)(2n+1)}{6} = \frac{1\times 2\times 3}{6} = 1.$$

Therefore S(1) is T.

(b)Inductive step. Assume S(k) is T, i.e. $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$, $k \in N^+$,

then we need to prove $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1))}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$.

Proof:

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)[(2k^2+k) + (6k+6)]}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Therefore, given S(k) is T, S(k+1) is T.

Based on (a) and (b), S(n) is true for any $n \in N^+$.