CSCI1323 Quiz 3 Key

(20 points) Prove the following open statement by mathematic induction.

$$\forall n \in Z^+, L_n = F_{n-1} + F_{n+1}$$

Solution – proof by the alternate form:

1. Basis step:

    S(1): $L_1 = F_{1-1} + F_{1+1}$

    Proof: $L_1 = 1$

    $F_{1-1} + F_{1+1} = F_0 + F_2 = 0 + 1 = 1$

    So S(1) is TRUE.

2. Inductive step:

    Assume S(1), …, S(k-1), S(k) are TRUE.

    S(k-1): $L_{k-1} = F_{k-1-1} + F_{k-1+1}$

    $L_{k-1} = F_{k-2} + F_k$

    S(k): $L_k = F_{k-1} + F_{k+1}$

    Show that S(k+1): $L_{k+1} = F_k + F_{k+2}$ is TRUE.

    Prove: $L_{k+1} = L_{k-1} + L_k$ By Lucas Number def'n.

    $= F_{k-2} + F_k + F_{k-1} + F_{k+1}$ By above assumptions.

    $= (F_{k-2} + F_{k-1}) + (F_k + F_{k+1})$ Rearranging.

    . $= F_k + F_{k+2}$ By Fibonacci def'n.

    So S(k+1) is TRUE.

Based on 1 and 2, S(n) is TRUE for any $n \in Z^+$.