Negate AΔB.

Solution:

We have

AΔB=$\{x | x \in A \cup B \wedge x \notin A \cap B\} = (A \cup B) - (A \cap B) = (A \cup B) \cap (\overline{A \cap B}), so$

| $\overline{A \Delta B} = \overline{(A \cup B) \cap (\overline{A \cap B})}$ | **Reasons** |
|---|---|
| $= (\overline{A \cup B}) \cup (\overline{\overline{A \cap B}})$ | DeMorgan's Law |
| $= (\overline{A \cup B}) \cup (A \cap B)$ | Law of Double Complement |
| $= (A \cap B) \cup (\overline{A \cup B})$ | Commutative Law of $\cup$ |
| $= (A \cap B) \cup (\overline{A} \cap \overline{B})$ | DeMorgan's Law |
| $= [(A \cap B) \cup \overline{A}] \cap [(A \cap B) \cup \overline{B}]$ | Distributive Law of $\cup$ *over* $\cap$ |
| $= [(A \cup \overline{A}) \cap (B \cup \overline{A})] \cap [(A \cup \overline{B}) \cap (B \cup \overline{B})]$ | Distributive Law of $\cup$ *over* $\cap$ |
| $= [\mathscr{U} \cap (B \cup \overline{A})] \cap [(A \cup \overline{B}) \cap \mathscr{U}]$ | Inverse Law |
| $= (B \cup \overline{A}) \cap (A \cup \overline{B})$ | Identity Law |
| $= (\overline{A} \cup B) \cap (A \cup \overline{B})$ | Commutative Law of $\cup$ |
| $= (\overline{A} \cup B) \cap (\overline{\overline{A} \cap B})$ | DeMorgan's Law |
| $= \overline{A}\, \Delta B$ | |
| $= (A \cup \overline{B}) \cap (\overline{A} \cup B)$ | Commutative Law of $\cap$ |
| $= (A \cup \overline{B}) \cap (A \cap \overline{B})$ | DeMorgan's Law |
| $= A\, \Delta \overline{B}$ | |