

Logical Equivalences and logical implications for quantified statement

For a prescribed universe and any open statements $p(x)$, $q(x)$:

$$\exists x[p(x) \wedge q(x)] \Rightarrow [\exists x p(x) \wedge \exists x q(x)]$$

$$\exists x[p(x) \vee q(x)] \Leftrightarrow [\exists x p(x) \vee \exists x q(x)]$$

$$\forall x[p(x) \wedge q(x)] \Leftrightarrow [\forall x p(x) \wedge \forall x q(x)]$$

$$[\forall x p(x) \vee \forall x q(x)] \Rightarrow \forall x[p(x) \vee q(x)] \quad \longrightarrow \text{Note these!}$$

$$\forall x [p(x) \wedge (q(x) \wedge r(x))] \Leftrightarrow \forall x [(p(x) \wedge q(x)) \wedge r(x)] \quad \text{Associate}$$

$$\forall x [p(x) \rightarrow q(x)] \Leftrightarrow \forall x [\neg p(x) \vee q(x)] \quad \text{Impli. Elimination}$$

$$\forall x \neg \neg p(x) \Leftrightarrow \forall x p(x) \quad \text{Double Negation}$$

$$\forall x \neg [p(x) \wedge q(x)] \Leftrightarrow \forall x [\neg p(x) \vee \neg q(x)] \quad \text{De Morgan}$$

$$\forall x \neg [p(x) \vee q(x)] \Leftrightarrow \forall x [\neg p(x) \wedge \neg q(x)] \quad \text{De Morgan}$$

Negate a quantified statement

$$\neg[\forall x p(x)] \Leftrightarrow \exists x \neg p(x)$$

$$\neg[\exists x p(x)] \Leftrightarrow \forall x \neg p(x)$$

$$\neg[\forall x \neg p(x)] \Leftrightarrow \exists x p(x)$$

$$\neg[\exists x \neg p(x)] \Leftrightarrow \forall x p(x)$$