HW6 First-Order Logic

(Due at the beginning of the class on Thursday, April 10)
In total of 25 points

Use your own paper, type your answers and show work as much as possible.

1. (4 points) Give a *reasonable* translation of the following natural language sentences into first-order logic.[1]

   a. "Every chicken hatched from an egg."
   b. "Superman flew over the tallest building in the world." (note: Don't just say *Tallest-Building* (x), use the function 'height' to describe the relationship among buildings).
   c. "There once was a man from Seattle, who didn't own any cattle." (hint: cattle is a property of many objects, not an object itself).
   d. "A cynic is a person who knows the price of everything and the value of nothing." (Oscar Wilde)

2. (5 points) Consider the following model for a first-order logic theory containing the constant *a*, the function f, and predicates P and *Q*: universe is *U*={1, 2}; denotations are $a = 1,\ f(1) = 2\ and\ f(2) = 1,$ extensions are $P = \{\langle 2 \rangle\}$ and $Q = \{\langle 1,1 \rangle, \langle 1,2 \rangle, \langle 2,2 \rangle\}$. Determine the truth value of the following sentences in the model. Be sure to show all of your work.

   a. $\forall X\ P(X) \rightarrow Q(f(X), a)$
   b. $\exists X\ P(f(X)) \wedge Q(X, f(a))$
   c. $\exists X\ \neg P(X) \wedge Q(X, a)$
   d. $\forall X \exists Y\ P(X) \wedge Q(X, Y)$
   e. $\neg \exists X \forall Y\ Q(f(Y), X)$

3. (2 points) Show that the following first-order sentence is satisfiable by giving a model that satisfies it (refer to question 2 about how to define the model).[2]

   $\forall X \forall Y\ P(X, Y) \rightarrow Q(Y), P(a, f(b)), \neg Q(a) \wedge \neg Q(b)$

4. (7 points) Determine whether or not the following pairs of predicates are unifiable. If they are, give there most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. Variables are in capital letters; constants are lowercase.

   a. $P(a, X, f(g(Y)))$ and $P(Z, f(Z), f(U))$
   b. $Q(f(a), g(X))$ and $Q(Y, Y)$
   c. $R(f(Y), Y, X$ and $R(X, f(a), f(V))$

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[1] State any assumptions you make about the meanings of words.
[2] In this problem, *X* and *Y* are variables, while *a* and *b* are constant terms.

d. $P(a, Y, f(X))$ and $P(X, f(b), f(b))$
e. $Q(g(f(a)), g(X), Z)$ and $Q(Y, Y, f(X))$
f. $P(a, X, g(f(f(a)), X))$ and $P(Z, f(Z), g(Y), f(Z)))$
g. $Q(f(a, a), Y, Z)$ and $Q(X, f(Z, Z), Y)$

5. (7 points) Using first-order rules of inference to prove that "there exists a vegetarian" from the following pieces of knowledge: anyone who does not eat meat is a vegetarian, tomatoes are not meat, carrots are not meat, and there is someone who eats only tomatoes and carrots. The initial sentences (premises) are translated into first-order logic for you below. The goal is to generate: $\exists X$ *vegetarian* (X). Be sure to explicitly label each new sentence with the one(s) it was derived from, along with the inference rule and any substitution used. (Hint: try universal instantiation, existential instantiation, implication elimination, and resolution.)

   1. $\forall P(\forall X eat(P, X) \rightarrow \neg meat(X)) \rightarrow vegetarian(P)$
   2. $\forall X tomato(X) \rightarrow \neg meat(X)$
   3. $\forall X carrot(X) \rightarrow \neg meat(X)$
   4. $\exists P \forall X eat(P, X) \rightarrow (tomato(X) \vee carrot(X))$