Administrivia

- Quiz 5 moved to next Monday.
- Homework 6 due date moved to this Friday.

Permutations and Combinations — Eliminating Duplicates

- In general it can be interesting to try to figure out how to “eliminate duplicates” — i.e., account for the fact that one way of counting things produces a lot of duplicate results.
- Example: How many ways can we rearrange the letters in the word “voodoo”? 
Permutations and Combinations With Repetitions

- Definitions of $P(n, r)$ and $C(n, r)$ specified "without repeats". What if we want to allow repeats?

- Permutations: How many ways can we choose an ordered sequence of $r$ things from $n$ possibilities, if we allow repeats? (Not too tough, right?)

- Combinations: How many ways can we choose an unordered collection of $r$ things from $n$ possibilities, if we allow repeats? This is trickier. We’ll use a clever idea from example 58.

Permutations and Combinations, More Examples

- Section 3.2, problem 25.
- Section 3.4 problem 31.
Probability — Equally-Likely Outcomes

- Basic definition: If \( S \) ("sample space") is a set of equally likely outcomes of some action (e.g., possible results of tossing a fair coin), and \( E \) ("event") is a subset of \( S \), then we define the probability of \( E \) as

\[
P(E) = \frac{|E|}{|S|}
\]

Examples: Sequences of coin tosses, 5-card "hands" chosen from 52-card deck, etc.

- Note that \( 0 \leq P(E) \leq 1 \). (Why?) When is \( P(E) = 0 \)? When is \( P(E) = 1 \)?

- Note that we can apply anything we know about sizes of sets. (E.g., if \( E_1 \) and \( E_2 \) are disjoint, what is \( P(E_1 \cup E_2) \) in terms of \( P(E_1) \) and \( P(E_2) \)?)

Minute Essay

- Given 20 words, how many 6-word phrases can you make up, if no repeated words are allowed? ("refrigerator magnet poetry")

Okay to express answers in terms of \( P(n, r) \) and/or \( C(n, r) \) or factorials.

- Suppose you select 6 marbles at random from a jar containing red, blue, yellow, and green marbles (at least 6 each). How many ways can this selection be made?
Minute Essay Answer

- Order matters here, so \( P(20, 6) \)
- Order doesn’t matter here, but repetitions are allowed, so this is a case of “combinations with repetitions”, so there are \( C(6 + 4 - 1, 6) = C(9, 6) \) ways to select.